Modeling Market Sentiment and Conditional Distribution of Stock Index Returns under GARCH Process

A final project submitted to the faculty of Claremont Graduate University in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics

by

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Abstract

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In finance, one of the greatest challenges is to measure investor sentiment correctly. A shortcoming of previous studies has been their failure to find an appropriate methodology which would define market sentiment correctly and use stock volatility to measure a direct relationship between sentiment and expected returns. Using survey-based measures of both individual and institutional investor sentiment, along with a set of macroeconomic variables, I employ a generalized autoregressive conditional heteroscedasticity specification, which employs not only the conditional volatility but also implied volatility (VIX) in the mean equation to test the impact of investor sentiment on stock returns. First, I find support for a negative relation between stock returns and implied volatility. Second, I find a positive and statistically significant relationship between changes in the sentiment Bull Ratio of both institutional and individual investors and S&P 500 excess returns in the following month. The estimation results also suggest that the opinions of institutional investors seem to relate to market data better than those of individual investors. In behavioral models, it is believed that investors' widespread optimism and pessimism can cause prices to deviate from their fundamental values, leading to temporary price corrections in the form of mean-reverting behavior when those expectations are not met. Using periodic *realized* market returns as anchors, I find a positive (negative) insignificant (significant) relationship between daily index returns and bullish (bearish) sentiment. These effects are stronger when the state of implied volatility is controlled as low, moderate, and high state.

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Chapter I

Investigating the Relationship between Implied Volatility and Stock Returns

I. Introduction

Since Markowitz (1952, 1959), who laid the groundwork for the Capital Asset Pricing Model (CAPM), forecasting volatility has become one of the great success stories in finance. Markowitz formulates the theory of optimal portfolio selection problem in terms of expected return and risk and argues that investors would optimally hold a mean-variance effect portfolio, a portfolio with the highest expected return for a given level of variance. A stock that is volatile is also considered higher risk because its performance may change quickly in either direction at any time. However, volatility is only one indicator of risk affecting a stock. Investors pay attention to volatility not because it is perceived as a mere measure of risk, but because they worry about unusual levels of excessive volatility, especially when observed fluctuations in a stock price do not seem to be accommodated by any related news about the firm's fundamentals. If this is the case, the stock price no longer might play its role as a signal about the true intrinsic value of a firm, Karolyi (2001). After Black and Scholes (1973) option pricing model, the use of implied volatility (versus historical volatility) in forecasting expected returns has become popular among researchers. For instance, Ederington and Guan (2002) find that most implied standard deviations averages calculated from several options for the S&P 500 futures options market forecast expected volatility better than naive time series models, concluding that "implied volatility has strong predictive power and generally subsumes the information in historical volatility" (p.29).

II. Understanding VIX

Because many investors hold more than one stock, it is more suitable to measure or pay attention to the volatility of an index. The VIX index was introduced by R. Whaley in 1993 for S&P 100 index. In 2003, CBOE together with Goldman Sachs, updated the VIX calculation based on S&P 500 index by averaging the weighted prices of the index's vanilla puts and calls (that are not exotic) over a wide range of strike prices.[1] Conventional wisdom is that VIX tends to trend in the very short-term, mean reverting over the short-intermediate term, and moves in cycles over the long-term. Examination of Figure 1 over the sample period from 2001:01 - 2011:02 reveals that VIX is low and stable between 2003 and 2008, after the dot-com crisis and before the financial meltdown of 2008. During the dot-com crisis, VIX usually remained between 20 and 40. Also observe that right around the time of the housing crisis, it remained mostly above 20 as well as reaching its all time high level of 80. Since May 2010, VIX has come down to its low 20 level again as the economy gets itself out of recession.

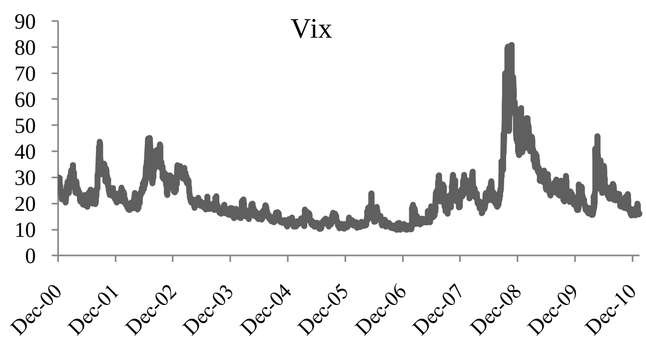
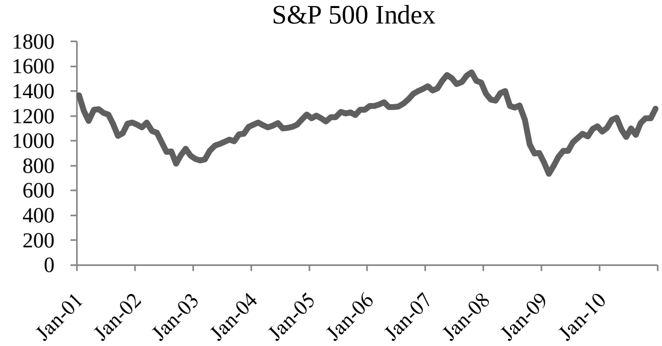


Figure 1

[1] VIX futures and options have become tradable assets in Exchange history. VIX represents the expected market volatility over the next 30 calendar days. It is the volatility of a *variance* swap[1]. For example, VIX 25 means that the market expects an *annualized* change of 25 percent in volatility over the next 30 days. This roughly corresponds to monthly 7.21 percent $\left(0.25/\sqrt{12}\right)$, means that the Index options are priced with the assumption of 68 percent likelihood (plus-minus one standard deviation) that the magnitude of the S&P 500 index 30-day return will be less than 7.21 percent (up or down).



One conclusion we can draw from *Figure 1* is that when VIX is low (between 2003-2007), the economy is more stable about its future direction such that we observe an upward slope in index returns (perhaps investors are confident and even over-optimistic during this period), whereas when we see short-lasting sudden spikes in VIX, we observe a downward slope in index (perhaps investors are starting to panic and there are big sell-offs in the market).

Table 1 shows VIX's descriptive statistics for the sample periods. To get a better idea about the data sample, I split it into three time periods: after the dot-com crisis, which ended around late 2003; during the housing bubble between 2004 and 2007; after the housing bubble and 2008 financial meltdown. We see very similar VIX statistics during both crises. VIX is more volatile during the crises with standard deviations 6.13 and 11.59, respectively, vs. 2.45 when the economy is booming during 2004-2007. *Figure 2* summaries VIX frequency table for the data sample. we observe almost 90 percent of VIX readings remained under 30 for the last decade.

Table 1 Descriptive Statistics	Full Data	Before 2004	Between 2004-2007	After July 2007
Mean	21.99	25.01	13.77	27.84
Median	20.19	23.45	13.28	24.15
Standard Deviation	9.93	6.13	2.45	11.59
Minimum	9.89	15.58	9.89	15.45
Maximum	80.86	45.08	25.16	80.86
Kurtosis	5.31	-0.06	1.50	3.68
Skewness	1.88	0.79	1.04	1.92
# of obs	2544	753	906	885

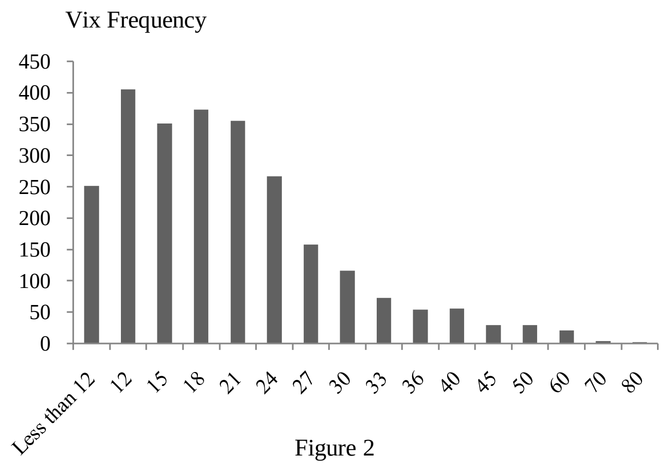


Figure 2

III. VIX as measuring Fear

VIX is often referred as the Investor Fear Gauge. Whaley et. al (1998) define VIX as a measure of investors' certainty (or uncertainty) regarding volatility. It is about fear of unknown such that the higher the VIX is, the greater the fear. That is, as VIX increases, the market becomes fearful and as it decreases, the market feels more confident about its future direction. One thing to keep in mind is that VIX does not cause volatility. It is an expectation of volatility and since volatility is directionless, so is VIX. For whether determining VIX is useful in forecasting S&P 500 daily returns (or vice versa), I do a Granger Causality Test. *Table 2* reports the test results. First define:

$$r_t = ln\left(\frac{SP_t}{SP_{t-1}}\right)$$

$$Vix_t = ln\left(\frac{Vix_t}{Vix_{t-1}}\right)$$

Table 2 *Granger Causality Test*

Part 1: For returns determining Vix

The following equation was estimated by OLS:

$$Vix_t = \mu + \sum_{i=1}^{L} \alpha_i Vix_{t-i} + \sum_{i=1}^{L} \beta_i r_{t-i} + \varepsilon_t$$

$H_0: \beta_1 = \cdots = \beta_L = 0$ (returns do not Granger cause Vix)

L = no. of lags	F-Statistics	P-value	R^2
1	0.1782	0.6730	0.009
2	0.0851	0.9184	0.015
3	0.0845	0.9685	0.017
4	0.0823	0.9878	0.017

Part 2: For Vix determining returns

The following equation was estimated by OLS:

$$r_t = \mu + \sum_{i=1}^{L} \alpha_i r_{t-i} + \sum_{i=1}^{L} \beta_i Vix_{t-i} + \varepsilon_t$$

$H_0: \beta_1 = \cdots = \beta_L = 0$ (Vix does not Granger cause returns)

L = no. of lags	F-Statistics	P-value	R^2
1	0.2426	0.6224	0.011
2	1.1711	0.3102	0.019
3	0.8266	0.4791	0.020
4	0.5816	0.6760	0.023

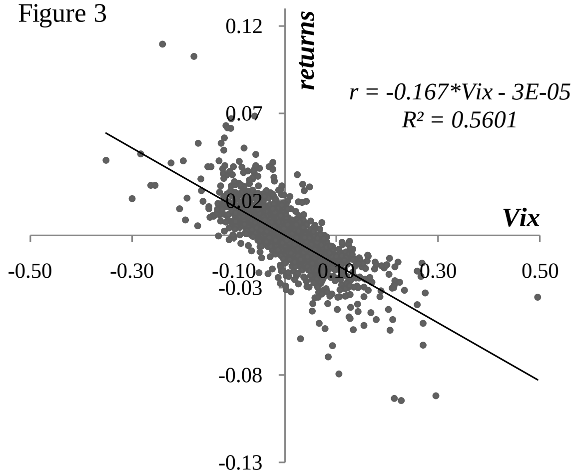
I find that the Granger causality test is inconclusive. None of the coefficients are significant in both tests. Therefore, we cannot reject the null hypothesis that VIX causes returns (or vice versa).

Apart from establishing a causation, the negative correlation between VIX and index returns has been well documented[2]. For example, Copeland and Copeland (1999) find that the changes in VIX today are correlated with the following return differences. *Figure 3* confirms this after I run the following regression.

$$r_t = \beta_0 + \beta_1 Vix_t + \varepsilon_t \qquad (1.1)$$

[2] See Whaley (1993, 2008) and Giot (2005).

Figure 3



The regression coefficient tells us that, on a given day, as Vix reading goes up by 10 percent, we expect the index return to go down on average by about 1.7 percent for that day. One explanation for this inverse relation is that options usually represent a form of insurance: high volatility implies higher option prices, so the insurance becomes expensive. When insurance becomes more expensive, investors demand higher rates of returns on stocks and this causes stock prices to fall. Another explanation is given by behavioral economic models, where they argue that investors appear to form beliefs based on psychological cognitive biases which can produce over/under reactions to fundamental and technical factors. When the implied volatility is increasing in the market place, people have a tendency to feel the pain or fear of regret at having made errors. In order to avoid this, they tend to alter their behaviors. In this environment, Shefrin and Statman (1985) claim that emotional investors are likely to sell past winners in order to postpone the regret associated with realizing a loss. They call this "*disposition effect*", selling their winning stocks too early and holding on to their losses too long.

IV. VIX under different Regimes

In this section, I investigate how sensitive the S&P 500 index returns are under different VIX readings. I first split VIX data into three categories: low, moderate, and high. The distribution of VIX, not reported here, has a mean of roughly 22 for the data sample. So I define a *moderate* regime as plus/minus half standard deviation from its mean, which corresponds to a range between 17.02 and 26.95. There are about 894 low regime, 1105 moderate regime, and 544 high regime observations for the sample period, corresponding 35%, 44%, and 21%, respectively indicating that it skewed left. The claim is that the slope of the regression line in Equation (1.1) should be steeper in high volatility state than it is in low or moderate state. The following regression is run to test this.

$$r_t = \beta_0 + \sum_{i=1}^{3} \beta_i D_i Vix_t + \varepsilon_t \qquad (1.2)$$

For each regime i, $D_i = 1$ if Vix_t *belongs to that regime*. $D_i = 0$, *otherwise*.

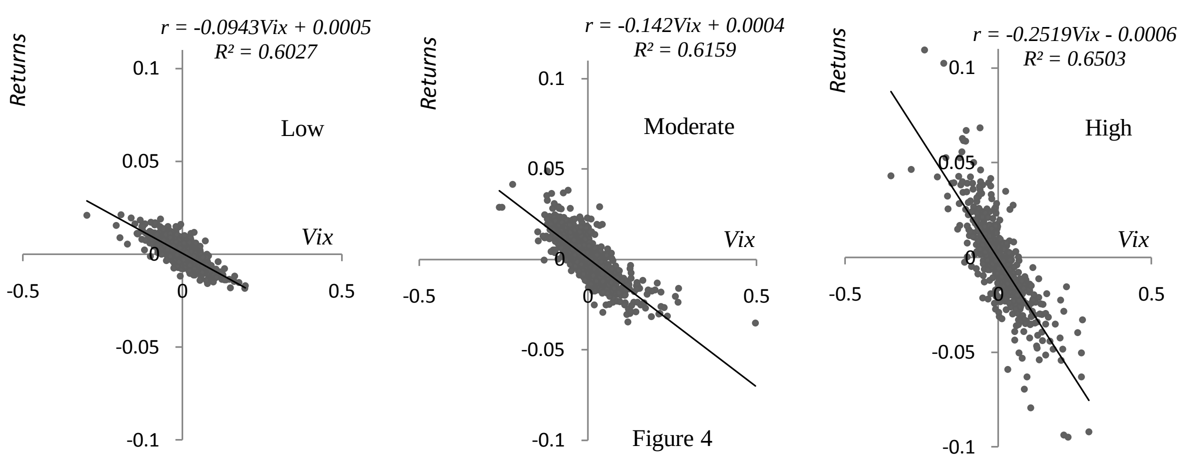


Figure 4

As claimed, *Figure 4* shows that stock index returns are more sensitive under high implied volatility state. DeBondt amd Thaler (1985) argue that investors are subject to waves of optimisim and pessimism of *herding* bias. Investors who communicate regularly tend to think and act similarly, perhaps they avoid being wrong in the group. In this tendency, they adopt the opinions and follow the behavior of the majority to feel safer and to avoid conflict. When VIX gets high, there is a greater panic and fear in the market place, which leads investors to sell off their holdings quicker than they would normally do. As a result, index returns drop sharply. We can apply the same logic to low VIX state, where investors do not expect big swings in stock prices and are more confident and optimistic about the direction of the market. When VIX goes up in this environment, they may not see it as a treat but instead as a temporary price correction. Therefore, they may choose to ignore it and react less by holding onto their positions longer.

V. Chapter Conclusion

In this chapter, I briefly argue that there is a strong relationship between implied volatility and index returns. VIX is believed to be a measure of investors' certainty regarding volatility. High volatility implies price turbulences (usually negative sharp drops in prices), whereas low volatility implies price stability (usually price-rallies and bubbles). In the next chapter, using survey-based data, I employ a generalized autoregressive conditional heteroscedasticity specification, where implied volatility (VIX) is exogenously added in the mean equation to test the impact of investor sentiment on stock returns.

Chapter II

Measuring Market Sentiment and Stock Index Returns

I. Literature Review

I.1. *Earlier Studies*

Over the years, numerous studies have been carried out on understanding how investors trade in the stock market. For many economists during the early period of the twentieth century, financial markets were still regarded as mere casinos. In standard finance, the expected utility theory (which focuses on the *level* of wealth) offers a representation of truly rational behavior under certainty. Using consumption discount models, Lucas (1978) claims that all investors have rational expectations and stock prices do fully reflect all available information.[3] He argues that if we can forecast agents' future consumption, rational asset prices may have a forecastable element related to consumption. However, Grossman and Shiller (1981) found that consumption discount models do not work very well unless the coefficient of relative risk aversion is set very high.

In contrast to the expected utility theory, Kahneman and Tversky (1979, 1992) offer the prospect theory, in which utility is defined over gains and losses (i.e. returns), relevant to a *reference point* rather than levels of wealth. They document behavioral systematic cognitive biases that are very common in human decision-making under uncertainty. They claim that people do not obey the normal axioms of the finance theory (expected utility; risk aversion problem; Bayesian updating; decision under uncertainty; and rational expectations) and they claim Bayes' rule is not an apt characterization of how individuals actually respond to new data. "…perhaps the most robust finding in the psychology of judgment is that people are overconfident…" (Kahneman and Tversky 1982).

Behavioral economists claim that understanding the behaviors of market participants is the key to understanding the market. These studies give psychological evidence explaining why

[3] This is the neoclassical version of the Efficient Market Hypothesis.

and how people make systematic errors in the way they think and claim that economists have ignored these biases in prior studies because they thought they would disappear when the stakes are high (LeRoy 1989). But today we see evidence that these biases are too important to ignore.[4] For example, *representative* bias is believed to lead investors to overreact to news while *conservative* bias leads investors to underreact to news.[5]

One of the major criticisms of behavioral finance is that people can find a story to fit the facts. LeRoy (1989) states that behavioral models are more successful in providing "after-the-fact" explanations for observed behavior than in generating testable predictions. Malkiel (2003) gives a good summary of why people should be skeptical of empirical results reported in behavioral finance literature. He believes that apparent patterns are extremely rare or too unstable to guarantee consistently superior investment results.

In many behavioral models inspired by DeLong, Shleifer, Summers, and Waldmann (DSSW (1990) hereafter), investors are of two types: professional investors who are sentiment-free and inexperienced investors who are prone to sentiment. In the effort of measuring investor sentiment and quantifying its effect, researchers mainly focused on Noise Trading theory, where individual (inexperienced) investors are blamed for creating excessive market volatility (noise). For example, according to Black (1986), the price of a stock reflects two things: information (that is observable and professional traders trade on) and noise (that is unobservable and the individual traders trade on). He claims that *noise* is the major reason for the use of decision rules that seem to violate the normal axioms of the finance theory. Shleifer and Summers (1990) argue that noise (rather than information) drives market participants' decisions in financial markets. DSSW (1990a, b) claim that noise traders falsely believe that they have unique information about the future price of a risky asset. Daniel et al. (1998) define noise trading as "variability in prices arising from unpredictable trading that seems unrelated to valid information".

By definition, inexperienced traders have different beliefs from other professional

[4] See Barberis and Thaler (2003) "Survey of a Behavioral Finance" for details. Several papers document such behavioral biases as *overconfidence* Daniel, Hirshleifer, and Subrahmanymean (1998); Barber and Odean (2001); Gervais and Odean (2001), as *overreaction* DeBondt and Thaler (1985), as *loss aversion* Kahneman and Tversky (1979); Shefrin and Statman (1985); Odean 1998, as *herding* Huberman and Regev (2001), as *mental accounting* Kahneman and Tversky (1981), and as *regret*.

[5] Griffin and Tversky (1992) state that in revising their forecast, people focus too much on the strength of the evidence and too little on its weight, relative to a rational Bayesian. Barberis, Shleifer, and Vishny (1998) explain *representativeness* as people's belief that they see patterns in random sequences.

investors, usually resulting from differences in processing information. One claim is that individual traders may lack the ability to distinguish noise from information; they think they are trading on information and do not know that they are trading on noise.

I.2. *Recent Studies*

In the literature, investor sentiment is defined as the general attitude towards the accumulation of a variety of fundamental and technical factors and takes three forms: bullish, bearish, and uncertain. Brown and Cliff (2004, 2005) suggest that stock market return and investor sentiment may act in a system. They employ two types of sentiment measures: one is direct measures from surveys, and the other is indirect measures from various market data such as market performance variables, derivative variables, and other sentiment measures such as closed-end fund discount rate, net purchases of mutual funds, proportion of fund assets held in cash, and number of IPOs. They find strong evidence that their sentiment measures co-move with the market in the long run (2-3 years). However, they find little evidence that sentiment has predictive power for near-term future stock returns.

Baker and Wurgler (2004, 2006, 2007) discuss cross-sectional differences in the time series of the stock returns. They claim that sentiment may differ across stocks and arbitrage possibilities may be different from one stock to another. They find evidence that investor sentiment affects the cross-section of stock returns and that the impacts are most profound on the stocks whose valuation is highly subjective and difficult to arbitrage. They suggest that the stocks most sensitive to shifts in investor sentiment are those companies that are younger, smaller, unprofitable, non-dividend paying, distressed, have extreme growth potential, and that have higher betas. These stocks will exhibit high sentiment beta.

Fisher and Statman (2000) were among the first to use survey-based measures of investor sentiment. They used three groups of investors: small investors (who are individuals taking *American Association Individual Investors* (AAII) surveys), semiprofessional investors (who are newsletter writers), and large investors (who are the institutions and Wall Street strategists). Using level of sentiment, they find that the sentiments of the three groups do not move together. Although both small investors and the semiprofessional investors are prone to be influenced by past returns, large investors are more careful and are not easily influenced by the past market price movements.

Verma and Soydemir (2009) employ survey-based data. Unlike the previous studies which usually treat sentiment as fully irrational, they focus on both rational and irrational components of investor sentiment. They use two surveys' data: individual investor survey from AAII and institutional investor survey from *Investor Intelligence* (II). They regress both these surveys on various rational factors including classical Fama-French factors and then, they define the fitted values from these regressions as the rational component of sentiment and the residuals as the irrational component of sentiment. Consistent with Brown and Cliff (2004), they find weak evidence between sentiment and market return in the short run but stronger evidence in the case of long-run. They claim that in times when irrational sentiment is high, noise trading can distort the price of risk, causing it to move away from the rates justified by economic fundamentals contributing to the formation of bandwagons and bubbles in the stock market.

II. Methodology for Measuring Investor Sentiment

II.1. *Surveys*

DeLong, Shleifer, Summer, and Waldmann (DSSW (1990)) study the effects of noise trading on equilibrium prices. They argue that noise traders act on non-fundamental information, which creates a systematic risk that is reflected on prices. They call this act *sentiment*. They claim that, when there are limits to arbitrage opportunities (in either direction) in the market, this risk created by shifts in sentiment forces prices to deviate from fundamental value, making prices unpredictable. DSSW (1990) predict that the direction and the magnitude of changes in sentiment are important elements in asset pricing. Their main focus is between changes in sentiment and returns. A sentiment measure might capture whether a group of investors are bullish or bearish for the stock market over a period of time. Studies using investor surveys as a direct measure of sentiment provide powerful empirical support for the hypothesis that stock prices are affected by investor sentiment.

I use two sentiment surveys' data: One, *American Association Individual Investors* (AAII). Two, *Investors Intelligent* (II). AAII is believed to represent *individual* investors. One argument is that individual investors might not act in line with their responses to surveys and AAII is a poor representation of individual investor sentiment. Using weekly survey data, Fisher and Statman (2000) claim that individual investors taking AAII surveys do follow their sentiment with investment action. They found a positive and statistically significant relationship between the monthly change in the sentiment and the monthly change in the stock allocation in their portfolios.[6] Another criticism is that AAII cannot fully represent the total market participants in the S&P 500 index. In fact, survey data from *Investors Intelligent* (II), which is designed to capture *institutional* sentiment, is a better representation of market participants. For this reason, I add II survey data into my analysis.

In Noise Trading theory, individual (inexperienced) investors are blamed for creating excessive market volatility (noise). Black (1986) claims that if there are no limits to arbitrage in the market, institutional investors will take positions against those noise traders (contrarian strategy). I investigate whether the correlation coefficient between the estimated sentiment for *institutional* and *individual* investors based on selected macro variables is, as claimed, negative or not. I find the coefficient to be 0.20. This can tell us two things: One, there are substantial limits to arbitrage in the market as Shleifer and Vishny (1997) claim, especially when fundamental traders manage other people's money, they may avoid taking extremely volatile "arbitrage positions" against noise traders because of high risk and the pressure from investors in the fund. Two, not all individual investors are inexperienced and noise traders. Perhaps due to improved information technology and telecommunication, many individual traders have direct access to information that those fundamentalist traders use and can follow how big funds are investing their money.

II.2. *Macroeconomic Variables*

As Fisher and Statman (2000) state, studies that use collected investor surveys can only explain the effects of *explicit* sentiment on stock returns. They argue that indicators of *implicit*

[6] They also find a negative and statistically significant relationship between the sentiment and future S&P 500 returns.

sentiment also need to be studied in order to understand the relationship between market sentiment and stock returns. In the effort to measure market sentiment, I use both *explicit* (the *observable* component of investor sentiment) and *implicit* (the *unobservable* component of investor sentiment). For the *explicit* sentiment, I use two surveys' monthly *Bull Ratios*. I then investigate the relationship between the log difference of *Bull Ratios,* called *Sent,* which is regressed on a set of macroeconomic variables, and the next month's stock returns. For the *implicit* sentiment, I use the residuals of those fitted values. For the definitions of *explicit* and *implicit* sentiment, my methodology is similar to the one that is adopted by Verma and Soydemir (2009). However, there are three major differences:

- Instead of the spread between the percentage of bullish investors and the percentage of bearish investors (Bull-Bear) from survey data, I use the log difference in *Bull Ratio.*
- Although some of the fundamental macroeconomic variables that are adopted by Verma and Soydemir (2009) are similar to the ones used here, the majority of my variables are different. I mainly focus on national economic data instead of market performance variables such as Fama-French factors.
- They use the Value at Risk (VaR) econometric approach to investigate the relationship between stock market returns and investor sentiment, whereas I use the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model.

II.2.1. *Selection Criteria for Macroeconomic Variables*

As Verma and Soydemir (2009) indicate, the ideal selection criteria for the macroeconomic indicators would be to select those that have small correlations with others so that each can represent a unique risk explaining investors *Bull Ratio.* They suggest using those that carry non-redundant information. But the nature of most macro variables is that they all somewhat have common components. In order to achieve clearer measures for their composite sentiment index, Baker and Wurgler (2007) try extracting the effects of the real business cycle from their selected variables by using Principal Components Analysis. They use six variables for their sentiment index.

Except for the fact that some variables are not available in monthly series (such as gross domestic product or government current expenditure, which are not selected for that reason), *Table 3* summarizes a list of variables for the selection process. The selection bias is an important issue in the literature. There can be many candidate variables used throughout the literature. I investigate 17 variables, but not necessarily all will be used. In my selection criteria, I intend to select those that are known to affect market sentiment (namely *Bull Ratio* here) and those that are found to be highly correlated with sentiment while they are weakly correlated with other selected variables. We can always add more variables or discard some of the variables from the list. Apart from personal favors or biases, I believe the power of this type of models should lie within its flexibility of choosing which variables to use and which not to use. It has to adapt to contemporary changes in economic activities and what investors are contemporarily paying attention to from one time to another. For example, there might be times when markets pay close attention to one particular (or a set of) variable(s) especially when it crosses over some important level such as happens in oil prices.

Table 3 **Macroeconomic Variables**

- Inflation Rate--CPI and PPI
- Real Disposable Personal Income (Inc)
- Default Spread (DS)--difference between BAA and AAA bonds
- Consumption Expenditure (CE2I)--over Disposable Income
- Funds Rate (FR)
- Yield Curve (YC) -- difference between 10-year T-bond and 3-month T-Bill (similar to Term Spread)
- Real Retail and Food Services Sales (RS)
- Jobs Opening in Manufacturing (Jobs)
- Inventory to Sales (I2S)
- Euro/Dollar Index (Euro)
- Trade Balance (TB)
- Housing Starts (H)-- Total New Privately Owned Housing Units Started
- Industrial Production (IP)-- in Manufacturing
- Consumer Sentiment Index (CSI)-- University of Michigan
- Unemployment Rate (UE)
- S&P 500 Volume (SPV)

Table 4 reports the correlation coefficients among these variables. Although most of the variables are correlated in the same direction with AAII and II, few variables have opposite signs such as *Inventory to Sales* and *Yield Curve*. First of all, there were not large differences in correlation coefficients for the sample period. And the conflict in signs can be ignored if the correlation coefficient is close to zero. For example, $corr(AAII, FR) = -0.01$ and $corr(II, FR) = 0.01$.

***Table 4* Cross Correlations**

	AAII	II	Jobs	Euro	CE2I	I2S	RS	TB	SPV	CSI	CPI	PPI	UE	FR	DS	YC	IP	H	Inc
AAII	1.00																		
II	0.36	1.00																	
Jobs	0.17	0.03	1.00																
Euro	0.04	0.20	-0.11	1.00															
CE2I	-0.07	-0.05	-0.05	-0.02	1.00														
I2S	0.14	-0.12	-0.17	-0.28	-0.20	1.00													
RS	0.00	0.22	0.06	0.06	0.33	-0.50	1.00												
TB	-0.16	-0.04	-0.04	-0.06	-0.11	0.14	0.01	1.00											
SPV	-0.06	-0.36	-0.03	-0.02	0.01	0.00	-0.10	0.03	1.00										
CSI	0.06	0.25	0.07	0.00	-0.09	0.04	0.11	0.04	-0.02	1.00									
CPI	-0.08	-0.13	0.05	0.30	0.16	-0.46	0.05	-0.42	0.21	-0.22	1.00								
PPI	-0.08	-0.08	0.02	0.39	0.03	-0.46	0.15	-0.39	0.21	-0.13	0.80	1.00							
UE	0.03	0.15	-0.18	0.03	-0.13	0.08	-0.07	0.25	0.04	0.02	-0.13	-0.20	1.00						
FR	-0.01	0.01	0.21	0.13	0.06	-0.17	0.13	-0.12	0.04	0.18	0.25	0.26	-0.24	1.00					
DS	0.13	0.01	-0.05	-0.26	-0.14	0.49	-0.27	0.03	-0.02	-0.10	-0.48	-0.46	0.05	-0.32	1.00				
YC	0.09	-0.08	-0.14	-0.16	0.04	0.00	-0.06	0.11	0.07	-0.11	0.07	0.03	0.07	-0.56	0.03	1.00			
IP	0.00	-0.09	0.29	0.05	0.06	-0.23	0.18	-0.16	-0.17	-0.07	0.08	0.19	-0.43	0.26	-0.06	-0.08	1.00		
H	-0.14	-0.08	0.10	0.02	-0.05	-0.28	0.17	-0.20	0.08	-0.14	0.07	0.11	-0.09	0.03	-0.12	-0.07	0.10	1.00	
Inc	0.05	0.14	0.07	0.00	-0.89	0.04	0.07	0.16	-0.07	0.15	-0.20	-0.04	0.10	-0.01	0.07	-0.06	0.04	0.07	1.00

III. Methodology for Measuring Index Returns

DSSW (1990) study the changes in sentiment and returns. Brown (1999) who also recognizes the importance of noise trading, however, focuses on the changes in sentiment and volatility. Lee, Jiang, and Indro (2002) argue that empirical tests which focused on the impact of sentiment either on expected return or volatility alone are misspecified. Using a *Generalized Autoregressive Conditional Heteroscedasticity* (GARCH) in-mean model (Bollerslev 1986, 1987; Engle et al. 1987), Lee et al. (2002) show that both the conditional volatility and excess returns are affected by investor sentiment. They examine the relationship between market volatility, excess returns, and investor sentiment for various market indices. Their results show that sentiment is a priced risk factor. More specifically, they find that excess returns are contemporaneously positively correlated with shifts in sentiment and the magnitude of bullish (bearish) changes in sentiment leads to downward (upward) revision in volatility and consequent higher (lower) future excess returns.

III.1. *Theory behind GARCH Models*

Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model is developed by Bollerslev (1986), which is an extension of the ARCH model, first introduced by Engle (1982) to explain the volatility of inflation rates. Numerous studies have examined the relationship between expected return and risk (not necessarily but often measured by conditional volatility, Lee et al. [2002]). Consistent with the behavioral economists' hypothesis that investors put more weight on recent past data, a simple GRACH in-mean setting allows the future values of mean and variance to be conditional on its past values so the errors are not constant, in the form that a series with some periods of low volatility tends to be followed by low volatility, and high volatility tends to be followed by high volatility. This is a well-known phenomenon called "volatility clustering", named by Mandelbrot (1963), and because it has a direct impact on returns, it is included in the mean equation[7]. Despite their sensitivity to some extreme data points, GARCH models are reviewed as more appropriate methodologies explaining financial data that exhibit excess kurtosis and tail thickness.

IV. Model

IV.1. *Measuring Investor Sentiment*

After determining what macro variables to use for each sentiment data, I regress *Bull Ratio* on those selected macro variables. *Sent* variable is calculated in the last week of each month representing a sentiment measure for the following month, whereas all macro variables are taken within the month they are released. Therefore, I write macro variables with *t-1* time subscript.

[7] Later, it will be replaced by the implied volatility (S&P 500's VIX Index).

$$Sent_t = \beta_0 + \sum_{i=1}^{10} \beta_i Macro_{it-1} + u_t$$

where β_i is the parameters to be estimated; u_t is the random error term. $Sent_t$ represents log-change *Bull Ratio*, the shift in sentiment at time t. $Macro_{it-1}$ is the set of national economic variables. The fitted values from this regression captured the *observable* component of investor sentiment and the residuals are assumed to capture the *unobservable* component. Both observable and unobservable components estimated here will be used to investigate monthly market excess returns.

IV.2. *Measuring Market Excess Returns*

The following analysis follows a general $GARCH(p,q)$-in-mean model with *m* number of autoregressive terms and no sentiment parameters. GARCH-in-mean models allow us to set conditional volatility (h_t) in the mean equation so that we can analyze the effects of volatility on stock returns. In the literature, different types of GARCH models are suggested in order to capture "leverage effect".[8] Consistent with Kahneman (1992) value function that is loss averse, which claims negative shocks have bigger impacts on prices then positive shocks, Glosten, Jagannathan, and Runkle (1993) (GJR-GARCH hereafter) suggest that these innovations (depending on their nature) have an asymmetric impact on market volatility.

$$r_t = \mu_{t-1} + \sum_{i=1}^{m} \alpha_i r_{t-i} + \delta h_t + \varepsilon_t \ ;$$

$$\varepsilon_t \sim (0, h_t) \ \ where \ \varepsilon_t = z_t \sqrt{h_t} \ and \ z_t \sim (0,1) \ ;$$

$$h_t = \omega + \sum_{i=1}^{p} \theta_i \varepsilon_{t-i}^2 + \sum_{i=1}^{u} \gamma_i I_{t-i} \varepsilon_{t-i}^2 + \sum_{i=1}^{q} \beta_i h_{t-i};$$

where r_t: Monthly excess returns for period *t*

[8] Leverage effect is defined as investors in forming their expectations of conditional volatility may perceive positive and negative shocks differently. Also see Bollerslev (2008) for more details explaining different types of (A)GARCH models.

μ_{t-1}: Mean of r_t conditional on past information
h_t: Variance
ε_t: Residuals
I_{t-i}: Dummy variable for negative news shocks, if $\varepsilon_{t-1} < 0$
m: Number of autoregressive terms
p: Number of ARCH terms
u: Number of news asymmetry terms
q: Number of GARCH terms
and $|\gamma_i| < 1$

A stationary solution exists if $\omega > 0$, and $\sum_i \theta_i K_i + \sum_i \beta_i < 1$, where $K_i = \mathrm{E}[|z| + \gamma_i z]^2$. Engle's (1982) ARCH model uses the normal distribution of residuals z_t. Using $\varepsilon_t = z_t\sqrt{h_t}$, the log-likelihood function of the normal distribution is given by:

$$lnL(\boldsymbol{\theta}) = \sum_{t=1}^{T} -\frac{1}{2}\left[\ln(2\pi) + \ln(h_t) + \frac{\varepsilon_t^2}{h_t}\right] = \sum_{t=1}^{T} -\frac{1}{2}[\ln(2\pi) + \ln(h_t) + z_t]$$

The full set of parameters $\boldsymbol{\theta}$ includes the parameters from the mean equation $(\mu, \alpha_{1:m}, \delta)$, from the variance equation $(\omega, \theta_{1:p}, \gamma_{1:u}, \beta_{1:q})$, and the distribution parameters from z_t in the case of a non-normal distribution function. Bollerslev (1987) proposed a standardized Student's t-distribution with $\lambda > 2$ degrees of freedom whose density is given by:

$$D(z_t; \lambda) = \frac{\Gamma\left(\frac{\lambda+1}{2}\right)}{\Gamma\left(\frac{\lambda}{2}\right)\sqrt{\pi(\lambda-2)}}\left(1 + \frac{z_t^2}{\lambda-2}\right)^{-(\lambda+1)/2}$$

where $\Gamma(\lambda) = \int_0^\infty e^{-x}x^{\lambda-1}dx$ is the gamma function and λ is the parameter measuring the tail thickness (Alberg, Shalit, and Yosef, 2006). The log-likelihood function for the Student's t-distribution is given by:

$$lnL(\boldsymbol{\theta}) = T\left(ln\Gamma\left(\frac{\lambda+1}{2}\right) - \ln\left(\frac{\lambda}{2}\right) - \frac{1}{2}\ln\left(\pi(\lambda-2)\right)\right) - \frac{1}{2}\sum_{t=1}^{T}\left((\ln(h_t) + (1+\lambda)ln\left(1+\frac{z_t^2}{\lambda-2}\right)\right)$$

where lower λ indicates flatter tails and as $\lambda \to \infty$, the distribution approaches to a normal distribution. Fernandez and Steel (1998) proposed another method to introduce skewness in any symmetric univariate distribution:

$$f(z^*|\xi) = \frac{2}{\xi + \frac{1}{\xi}}\left[g\left(\frac{z^*}{\xi}\right)1_{\{z^*>0\}} + g(z^*\xi)1_{\{z^*<0\}}\right]$$

where $g(.)$ is the Student distribution and $f(z^*|\xi)$ is a unimodal density with an asymmetric skewness parameter $\xi > 0$ such that if the r^{th} order moment of $g(.)$ exists, the skewed distribution $f(z^*|\xi)$ has a finite r^{th} moment of $E(z^{*r}|\xi) = M_r \frac{\xi^{r+1} + \frac{(-1)^r}{\xi^{r+1}}}{\xi + \frac{1}{\xi}}$ (Lambert, Laurent, and Veredas, 2007) and the first two moments are given by:

$$E(z^*|\xi) = M_1\left(\xi - \frac{1}{\xi}\right) = \mu$$

$$Var(z^*|\xi) = (M_2 - M_1^2)\left(\xi^2 + \frac{1}{\xi^2}\right) + 2M_1^2 - M_2 = h$$

Transforming $z \to \frac{z-\mu}{\sqrt{h}}$ yields skewed distributions, where the parameter μ can be interpreted as the mean or location parameter and the parameter $\sqrt{h}$ can be interpreted as the standard deviation or the dispersion parameter. [9] Lambert and Laurent (2001) extended the skewed Student-t distribution where the random variable $z_t = \frac{z_t^* - \mu}{\sqrt{h}}$ is said to be skewed Student-t, $SKST(0,1,\xi,\lambda)$ with $\lambda > 2$ and $\xi > 0$, if:

[9] See Lambert and Laurent (2001a,b); Wurtz, Chalabi, and Luksan (2009) for more details. I also considered other distributions such as normal and generalized error distribution (ged). Consistent with the claim, the results from these distributions were inferior relative to skewed student-t distribution.

$$\mu(\xi,\lambda) = \frac{\Gamma\left(\frac{\lambda-1}{2}\right)\sqrt{\lambda-2}}{\sqrt{\pi}\Gamma\left(\frac{\lambda}{2}\right)}\left(\xi - \frac{1}{\xi}\right);$$

$$h(\xi,\lambda) = \left(\xi^2 + \frac{1}{\xi^2} - 1\right) - \mu^2;$$

its density is given by:

$$f(z_t|\xi,\lambda) = \begin{cases} \left(\frac{2}{\xi+\frac{1}{\xi}}\right)\sqrt{h} * g\left[\xi * \left(\sqrt{h}z_t + \mu \mid \lambda\right)\right] & \text{if } \ z_t < -\frac{\mu}{\sqrt{h}} \\ \left(\frac{2}{\xi+\frac{1}{\xi}}\right)\sqrt{h} * g\left[\left(\frac{\sqrt{h}z_t+\mu}{\xi} \ \mid \lambda\right)\right] & \text{if } \ z_t \geq -\frac{\mu}{\sqrt{h}} \end{cases};$$

such that

$$I_t = \begin{cases} 1, \ if \ z_t \geq -\frac{\mu}{\sqrt{h}} \\ -1, \ if \ z_t < -\frac{\mu}{\sqrt{h}} \end{cases};$$

The log-likelihood function for the skewed Student's t-distribution is given by:

$$lnL(\boldsymbol{\theta}) = T\left(ln\Gamma\left(\frac{\lambda+1}{2}\right) - \ln\left(\frac{\lambda}{2}\right) - \frac{1}{2}\ln\left(\pi(\lambda-2)\right) + \ln\left(\frac{2}{\xi+\frac{1}{\xi}}\right) + \ln\left(\sqrt{h}\right)\right) - \frac{1}{2}\sum_{t=1}^{T}\left(\ln(h_t) + (1+\lambda)ln\left(1 + \left(\frac{\sqrt{h}z_t+\mu}{\lambda-2}\right)\xi^{I_t}\right)\right)$$

IV.2.1. *Model 1:* (*Base Model without Sentiment)*

In benchmark, I exclude sentiment as an explanatory variable in the mean equation and start my analysis using a simple GARCH(1,1) model, where $\delta = 0$, $p = 1$, $q = 1$, $u = 0$, and $m = 2$.[10]

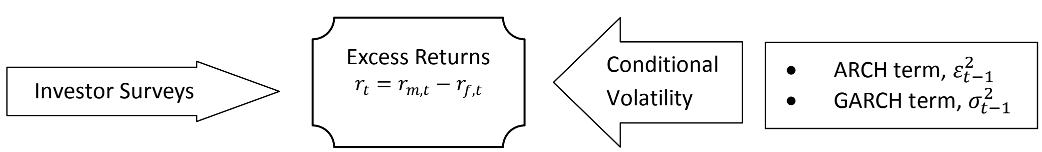
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim (0, h_t);$$

$$h_t = \omega + \theta \varepsilon_{t-1}^2 + \beta h_{t-1}.$$

IV.3 *Measuring Market Excess Returns using Investor Sentiment*

IV.3.1. *Model 2: (Adding Sentiment Parameters directly)*

In this setting, I investigate Fisher and Statman's (2000) findings, where using multiple regressions, they found no signs of a meaningful relationship between *change* in sentiment and S&P 500 returns in the next period[11]. They argue that collected investor surveys can explain the effects of sentiment on stock returns. Different from their technique, I use GARCH-in-mean models. The following diagram summarizes the methodology that is adopted here.



[10] I also tested the model with AR(1) and GJR innovations. The results were either similar or inferior. Moreover, Lee et al. (2002) suggest that market volatility tends to be higher in high inflation periods. So I add R_{ft} in the conditional volatility. Again, adding risk-free rate did not improve the test results. One explanation may be that the U.S. inflation rate was low, remained under four percent, during the last ten years, which had no impact on conditional volatility.

[11] They used weekly series.

I use both *individual* and *institutional* investors surveys, separately and together. There are a total of three models to estimate. [12]

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + bSent_t^{Indv} + \varepsilon_t \; ;$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + bSent_t^{Inst} + \varepsilon_t \; ;$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + b_1 Sent_t^{Indv} + b_2 Sent_t^{Inst} + \varepsilon_t \; ;$$
$$h_t = \omega + \theta \varepsilon_{t-1}^2 + \beta h_{t-1};$$

where $Sent_t$ represents change in sentiment, more specifically, $Sent_t = \ln \left(\frac{Bull\ Ratio_t}{Bull\ Ratio_{t-1}}\right)$.

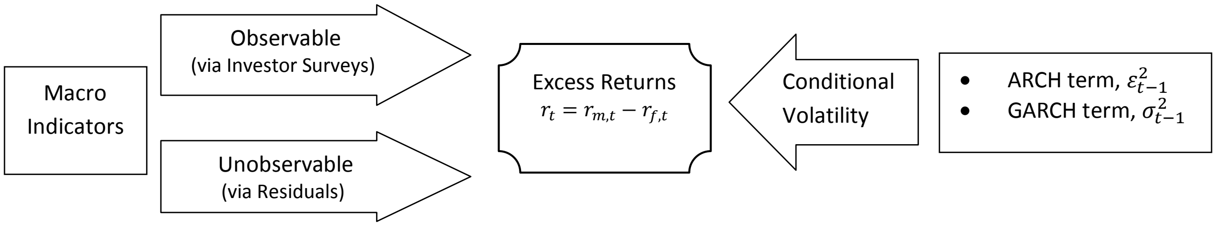
IV.3.2. *Model 3: (Adding Sentiment Parameters after Regressing them on Selected Macro Variables)*

It is claimed that when investors are bullish about the economy, stock returns, on average, go up in the next month. Few questions arise from this relationship:

- What are the factors that affect shifts in investors sentiment?
- How do these factors affect different groups of investors?
- Are there other factors that affect each group of investors differently?

If we were to assume that *individual* investors and *institutional* investors are affected by various factors that are not necessarily dependent on each other, then we would expect that both groups have a small correlation coefficient. In Section 2.3, I found the correlation coefficient to be 0.36, which indicates that although there are similar variables affecting both groups of investors sentiment in the same direction, there may be other variables that affect only one group of investors but not the other. Therefore, in this section, I add the estimated sentiment parameters as explanatory variables, which are obtained from the selected macro variables, in the mean equation and then investigate their effects on S&P 500 excess returns in the following month.

[12] I also included the squared lagged shifts in observable and unobservable component in the conditional volatility as in Chen et al. (2008) and Lee et al. (2002), the results were insignificant.



There are a total of four models to estimate.

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + b_1 \widehat{Sent}_t^{Indv} + b_2 u_t^{Indv} + \varepsilon_t \; ;$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + b_1 \widehat{Sent}_t^{Inst} + b_2 u_t^{Inst} + \varepsilon_t \; ;$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + b_1 \widehat{Sent}_t^{Indv} + b_2 \widehat{Sent}_t^{Inst} + \varepsilon_t \; ;$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + b_1 \widehat{Sent}_t^{Indv} + b_2 \widehat{Sent}_t^{Inst} + b_3 u_t^{Indv} + b_4 u_t^{Inst} + \varepsilon_t \; ;$$
$$h_t = \omega + \theta \varepsilon_{t-1}^2 + \beta h_{t-1} \; ;$$

where $\widehat{Sent}_t$ represents the fitted values; u_t is the residuals from $Sent_t = \beta_0 + \sum_{i=1}^{10} \beta_i Macro_{it-1} + u_t$; $\widehat{Sent}_t = \hat{\beta}_0 + \sum_{i=1}^{10} \hat{\beta}_i Macro_{it-1}$; and $u_t = Sent_t - \widehat{Sent}_t$.

Sent captures shifts in investor sentiment (*Bull Ratio*) measured by surveys. Verma and Soydemir (2009) define *Sent-hat* as the rational component of investor sentiment and u_t as the irrational component of investor sentiment. However, given that AAII-*Sent-hat* only captures about 15 percent and II-*Sent-hat* captures about 25 percent of variations on *Bull Ratio,* which suggests there exist other factors, if not more important, at least as important as those selected macro variables, I believe interpreting these variables as *rational* and *irrational* is inappropriate.[13] Instead, inspired by Fisher and Statman (2000), I interpret *Sent-hat* as the *observable* component of investor sentiment and u_t as the *unobservable* component of investor sentiment. I investigate whether *both* observable and unobservable components have effects on excess returns.

[13] They also report R-squared values of 0.30 and 0.16 for individual and institutional investors, respectively.

IV.4. *Measuring Market Excess Returns and Sentiment under Implied Volatility*

The CAPM claims that investors should be awarded for taking extra risk in the market. As in Lee et al. (2002), risk is, not necessarily but often, measured by conditional volatility in the GARCH-in-mean setting. They interpret the sign of conditional volatility, h_t in the mean equation as price for time-varying risk so that the positive coefficient suggests that investors are compensated for taking more risk, whereas the negative coefficient suggests that investors are penalized for the extra risk they take. French et. al (1987) estimate GARCH-in-mean models on the daily excess returns of the S&P composite index for the period 1928 to 1984. They use both the conditional variance and the conditional standard deviation specification and provide evidence for a significant positive relationship between excess returns and risk. They claim that their results should support the CAPM's hypothesis.[14] In this section, instead of adding conditional variance in the mean equation (GARCH-in-mean) and arguing that it represents the time-varying risk associated with changes in volatility, I propose a different approach: adding *implied* volatility in the mean equation. Given that a stock price of a firm today is calculated based on the firm's future earnings, *implied* volatility, if desired to be used as a measure of risk, should be a more appropriate measure of risk than *past* volatility because implied volatility measures the uncertainty associated with *future* expectations. In Chapter I, I showed that the relationship between stock returns and *implied* volatility is negative. Therefore, I expect the relationship between excess returns and risk, associated with implied volatility, to be negative. There are a total of eight models to estimate.

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + \varepsilon_t \;;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + bSent_t^{Indv} + \varepsilon_t \;;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + bSent_t^{Inst} + \varepsilon_t \;;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + b_1 Sent_t^{Indv} + b_2 Sent_t^{Inst} + \varepsilon_t \;;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + b_1 \widehat{Sent_t^{Indv}} + b_2 u_t^{Indv} + \varepsilon_t \;;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + b_1 \widehat{Sent_t^{Inst}} + b_2 u_t^{Inst} + \varepsilon_t \;;$$

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[14]The CAPM assumes that the variance of returns is a good measure of risk and returns are normally distributed.

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + b_1 \widehat{Sent}_t^{Indv} + b_2 \widehat{Sent}_t^{Inst} + \varepsilon_t \ ;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + b_1 \widehat{Sent}_t^{Indv} + b_2 \widehat{Sent}_t^{Inst} + b_3 u_t^{Indv} + b_4 u_t^{Inst} + \varepsilon_t \ ;$$

$$h_t = \omega + \theta \varepsilon_{t-1}^2 + \beta h_{t-1} \ ;$$

where Vix_t represents $logVix_t = ln\left({Vix_t}/{Vix_{t-1}}\right)$.[15]

IV.5. *Measuring Market Returns and Sentiment using Weekly Series*

Given that GARCH models are more appropriate for high frequency data series, in this section, I repeat the methodology adopted as in Section IV.1 and 2 by using *weekly* series. Because the selected macro variables are announced monthly, we have no way of re-testing the methodology in the section IV.3 for the weekly series. Moreover, when the high frequent series is used, negative sign bias of the residuals is in present. So I use GJR-innovation in the conditional variance. There are a total of eight models to estimate.

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + \varepsilon_t \ ;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + bSent_t^{Indv} + \varepsilon_t \ ;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + bSent_t^{Inst} + \varepsilon_t \ ;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + b_1 Sent_t^{Indv} + b_2 Sent_t^{Inst} + \varepsilon_t \ ;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + \varepsilon_t \ ;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + bSent_t^{Indv} + \varepsilon_t \ ;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + bSent_t^{Inst} + \varepsilon_t \ ;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + b_1 Sent_t^{Indv} + b_2 Sent_t^{Inst} + \varepsilon_t \ ;$$

$$h_t = \omega + \theta \varepsilon_{t-1}^2 + \gamma I_{t-1} \varepsilon_{t-1}^2 + \beta h_{t-1} \ .$$

[15] I also used various models (not reported here) with AR(1), GJR-innovation, and delta Vix_t, where it is defined as $\Delta Vix_t = Vix_t - Vix_{t-1}$. Again, the test results were either the same or inferior.

V. Data, Descriptive Statistics, and Estimation Results

V.1. *Survey Data*

Both surveys are taken weekly. For example, AAII asks individual Investors for forecasts of the stock market for the next six months. Investors responding to this survey have three opinions: they are bullish, bearish, or neutral. Each week *Investors Intelligence* surveys approximately 150 market newsletter writers. They take this survey on Friday and release the results to the media the following Wednesday. They both begin from January 2001 until December 2010. In order to convert weekly series into monthly data, I use the percentage of bullish investors in the last week of each month as a measure of investor sentiment. [16] I then calculate a *Bull Ratio* as following:

$$Bull\ Ratio_t = \frac{Number\ of\ Bullish}{Number\ of\ Bullish + Number\ of\ Bearish}$$

I adjust this ratio by taking the log difference and call it *Sent.*

$$Sent_t = \ln\left(\frac{Bull\ Ratio_t}{Bull\ Ratio_{t-1}}\right)$$

Table 5 provides descriptive statistics of *Bull Ratio* and *Sent* for both surveys.

[16] I also use the percentage of bullish investors in each week and calculate the monthly average as a measure of investor sentiment. Results were inferior.

Table 5	AAII Bull Ratio	II Bull Ratio	AAII *Sent*	II *Sent*
Mean	54.72%	62.56%	0.0022	0.0016
Median	54.05%	64.14%	0.0049	-0.0075
Std. Dev.	13.43%	9.24%	0.2686	0.1188
Variance	1.80%	0.85%	0.0721	0.0141
Minimum	29.54%	31.82%	-0.6138	-0.3488
Maximum	89.29%	76.08%	0.6637	0.4520
Kurtosis	-0.7912	0.4893	0.0274	1.8079
Skewness	0.2778	-0.8728	0.0365	0.3166
# of obs	120	120	120	120

For the sample period, *Bull Ratios* of AAII and II have a mean of 55% and 62.5%, respectively indicating that both groups of investors were generally optimistic about the economy with a standard deviation of roughly 13% and 9% during the sample period. Higher mean and lower standard deviation values of institutional investors suggest that they were more optimistic about the economy and more confident about their opinions than those individual investors. The mean of *Sent* is almost zero for both groups with a standard deviation of 27% and about 12%, respectively confirming the idea that institutional investors are firmer in their opinions. It may also suggest that although *Sent* fluctuates often, it trends around the mean (mean revert). Normal distributions produce a kurtosis statistic of about zero.[17] Negative excess kurtosis indicates a relatively flat distribution, whereas positive excess kurtosis indicates a relatively peaked distribution. Although both *Sent* measures have positive kurtoses, changes in II-*Bull Ratio* have higher peaked distribution than that of AAII-*Bull Ratio*. Positive skewness in both datasets indicates a distribution with an asymmetric tail extending towards more positive values. Normal distributions produce a skewness statistic of about zero. Given that both *Sent* measures have skewness less than ½, we can claim that their distributions are close to normal or moderately skewed.

In *Figure 5,* we see two interesting consequential downward trends in *Bull Ratios* in two time periods for both groups of investors: 2003-2006 and 2007-2009. The period from mid 2003 until 2006 corresponds to the housing-bubble period, where S&P 500 index had an upward trend corresponding to a low VIX. Individual investors start this period with high optimism and their optimism declines rapidly as the bubble peaks, whereas, although they remained optimistic,

[17] The "kurtosis" reported by Excel is actually the excess kurtosis, which is simply kurtosis-3.

institutional investors' optimism did not die out so rapidly. This is consistent with Shefrin and Statman's (1985) "*disposition effect*", where individual investors are more emotional than professional investors and likely to sell their winning stocks too early in order to postpone the regret associated with realizing a loss. The second downward trend corresponds to the period of 2008 financial meltdown, this time where we see a rapid decline in institutional investors' *Bull Ratio*, relatively speaking. Although they do not get as emotional as individual investors, institutional investors show a great degree of pessimism during the housing crisis.

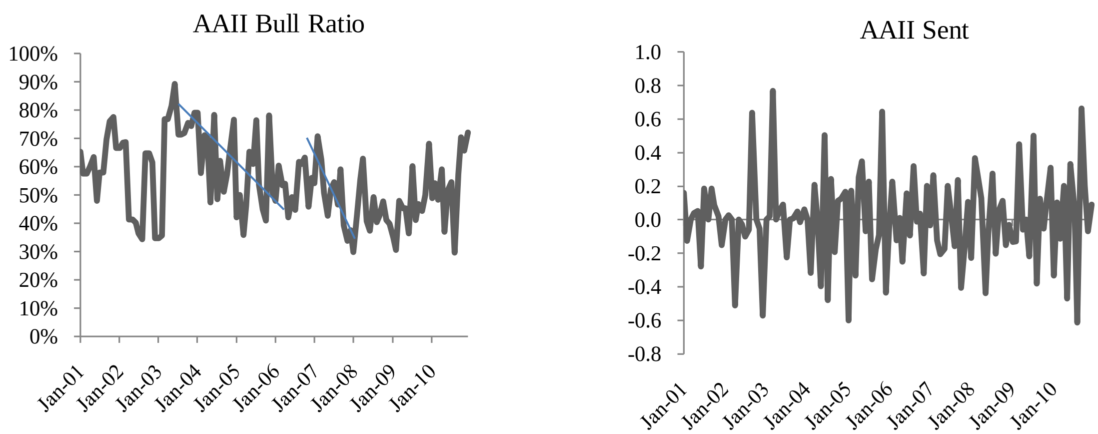
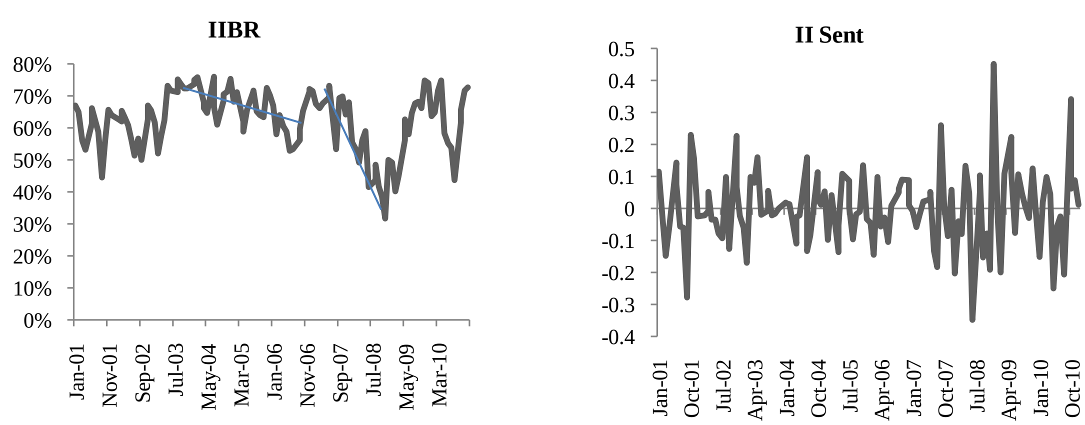


Figure 5



V.2. *S&P 500 Stock Index Returns*

The monthly adjusted closing prices of the S&P 500 index data series from January 2001 until December 2010 are used for the analysis. The market return is defined as:

$$r_{m,t} = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

I use the monthly excess market returns defined as the difference between the monthly index return and the monthly risk free rate. I use the three-month Treasury bill divided for each month as the risk free rate which is available on Federal Reserve Bank of St. Louis's online data library: $r_t = r_{m,t} - r_{f,t}$. Descriptive statistics for the monthly market returns are summarized in *Table 6*.

Table 6	S&P 500 Index	S&P 500 Returns
Mean	1157	-0.04%
Median	1148	0.81%
Std. Dev	182	4.81%
Variance	33003	0.0023
Minimum	735	-18.56%
Maximum	1549	8.98%
Kurtosis	-0.445	1.251
Skewness	0.035	-0.820
# of obs	120	120

For the sample period, the S&P 500 average monthly log returns was about -0.04 percent with roughly 4.8 percent standard deviation. Interestingly, there is a big size-gap between the highest and the lowest monthly returns. The worst month for the index performance was about -18.5 percent (during the 2008 housing crisis), whereas in its best month the index performed slightly under 9 percent during the sample period. Although the market has been more sensitive to negative news than positive news, the median remains about 0.8 percent, indicating that downward trends are sharp and quick, while upward trends are modest but long-lasting. Sample kurtosis is greater than one, indicating that the returns have a peaked distribution. From the skewness, negative excess skewness is observed, leading to high Jarque-Bera statistics indicating non-normality.

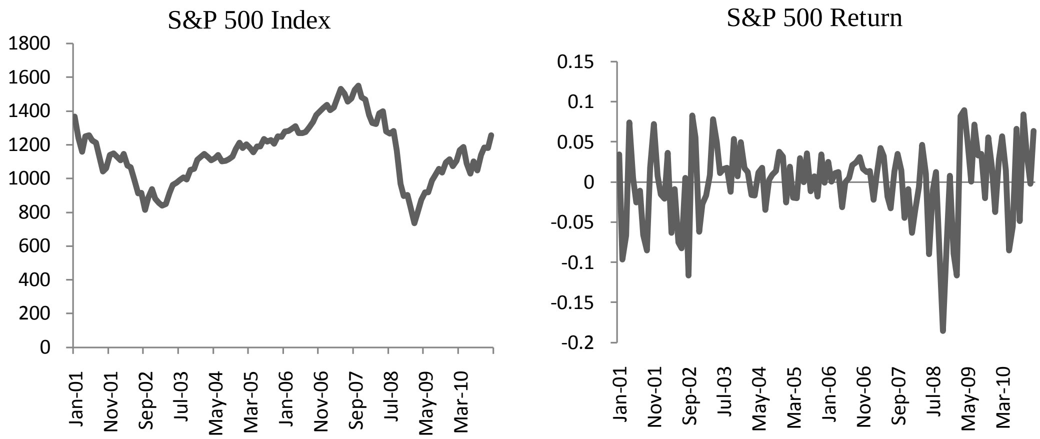


Figure 6

Figure 6 confirms those statistics in *Table 6*. We see two dips: One is around October 2002 following the 2000-dot-com bubble and the other one is around March 2009 following the 2008-housing crisis. During the dot-com crisis the index lost nearly 50 percent of its value in the following subsequent 3 years and it took nearly 5 years to reach back to where it was before the crisis. The impact of the 2008-housing crisis on the index was more severe. It lost about 56 percent of its value in slightly less than two years and it has been recovering slowly since as we speak. We also observe that index returns were mostly positive and less volatile fluctuating roughly plus-minus five percent during 2003-2007, consistent with the hypothesis that VIX is low during an expansion period.

V.3. *Estimations Results for Sentiment Measures*

V.3.1. *Individual Investor Sentiment*

I use ten selected macro variables to estimate the coefficients for individual investors survey. *Table 7* reports summary statistics for the estimated coefficients.

Table 7	*Individual Investor Sentiment (**AAII**)*				
	Estimate	Std. Error	t-value	Pr(>|t|)	
(Intercept)	0.0322	0.0289	1.114	0.2677	
Disposable Income	-0.0022	0.0012	-1.800	0.0746	*
Retail Sales	0.0657	0.0264	2.490	0.0143	**
PPI	-0.0278	0.0123	-2.259	0.0259	**
YC	0.1999	0.0855	2.339	0.0212	**
Euro Dollar	1.6920	0.8086	2.093	0.0387	**
Job Openings in Manufacture	0.0025	0.0009	2.808	0.0059	***
Inventory to Sales	3.3100	1.8220	1.816	0.0721	*
Expenditure to Income	-26.5300	12.9700	-2.045	0.0432	**
Trade Balance	-0.0251	0.0077	-3.273	0.0014	***
Housing	-0.0005	0.0002	-2.224	0.0282	**
R-squared	22.05%				
Adjusted R-squared	14.9%				
F-statistic	3.084				
Observation	119				

*Significant Codes: 0.01 '***' 0.05 '**' 0.10 '*'*

As *individual* investors observe a positive outlook for the economy, we expect the *Bull Ratio* to go up. The sign of economic stability and growth can ignite bullishness in the market. This period usually corresponds to an economic expansion, where businesses hire more workers that create more disposable income, which in return increases personal investment and spending that help business sales go up. Variables positively associated with changes in the *Bull Ratio* which are statistically significant include *Retail Sales*, *Job Openings in Manufacturing*, *Yield Curve*, *Euro Dollar*, and *Inventory to Sales*. On the other hand, *Disposable Income*, *Producer Price Index*, *Trade Balance*, *Expenditure to Income*, and *Housing* are statistically significant and negatively associated with changes in the *Bull Ratio*. I regard a negative sign of *Expenditure to Income* as a risk associated with people spending irresponsibly. When people spend more than what they are earning, it makes investors more pessimistic. I also regard an unexpected negative sign of *Disposable Income* as something to do with its colinearity with *Expenditure to Income*. They both are highly correlated.

V.3.2. *Institutional Investor Sentiment*

I also use ten selected macro variables (not necessarily the same chosen for individual investors) to estimate the coefficients for institutional investors survey. *Table 8* reports summary statistics for the estimated coefficients employed. Notice that some of the variables selected here are different from those used to measure individual investor sentiment.

Table 8	*Institutional Investor Sentiment (**II**)*				
	Estimate	Std. Error	t-value	Pr(>|t|)	
(Intercept)	-0.0047	0.0101	-0.4620	0.6452	
Disposable Income	0.0001	0.0001	0.7850	0.4344	
Retail Sales	0.0101	0.0056	1.7980	0.0749	*
Inventory to Sales	-0.9183	0.7520	-1.2210	0.2246	
Default Spread	0.1213	0.0713	1.7020	0.0917	*
Euro Dollar	0.7039	0.3080	2.2850	0.0242	**
Industrial Production	-0.0281	0.0157	-1.7860	0.0768	*
Unemployment	0.0550	0.0612	0.8980	0.3713	
Consumer Confidence	0.0056	0.0022	2.5450	0.0123	**
Housing	-0.0001	0.0001	-0.6510	0.5163	
SP500 Volume	-0.0765	0.0175	-4.3830	0.0000	***
R-squared	31.56%				
Adjusted R-squared	25.28%				
F-statistic	5.026				
Observation	119				

*Significant Codes: 0.01 '***' 0.05 '**' 0.10 '*'*

First of all, these selected variables explaining changes in institutional investors' *Bull Ratio* do a better job than those used for individual investors. Adjusted R-squared increased substantially. Although the coefficients of *Disposable Income* and *Inventory to Sales* are insignificant, they have the expected correct signs here. Although they both have smaller coefficients, both *Retail Sales* and *Euro Dollar* have the same positive and significant coefficients here as well. *Default Spread* has the similar positive effect but is significant here. I use four new variables measuring institutional sentiment: *Consumer Confidence Index* (positive-and-significant), *SP500 Volume* (negative-and-significant), Industrial Production (negative-and-significant), and *Unemployment Rate* (positive-and-insignificant). The Index volume has the most significant coefficient estimate. High volume can mean high volatility in the market place, perhaps an indication of block selloffs and decreases in bullishness. Although we would expect a negative sign for the unemployment rate, its coefficient is statistically insignificant. Only *Industrial Production* whose coefficient is significant came out odd here. We would expect a positive sign for it.

V.4. *Empirical Results for Market Returns*

V.4.1. *GARCH-in-mean for Monthly Series (with Conditional Volatility)*

I estimate the following equations using the GARCH-in-mean model with the skewed Student's t-distribution.

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + \varepsilon_t \qquad \textit{(Model 1)}$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + bSent_t^{Indv} + \varepsilon_t \qquad \textit{(Model 2.a)}$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + bSent_t^{Inst} + \varepsilon_t \qquad \textit{(Model 2.b)}$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + b_1 Sent_t^{Indv} + b_2 Sent_t^{Inst} + \varepsilon_t \qquad \textit{(Model 2.c)}$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + b_1 \widehat{Sent_t^{Indv}} + b_2 u_t^{Indv} + \varepsilon_t \qquad \textit{(Model 3.a)}$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + b_1 \widehat{Sent_t^{Inst}} + b_2 u_t^{Inst} + \varepsilon_t \qquad \textit{(Model 3.b)}$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + b_1 \widehat{Sent_t^{Indv}} + b_2 \widehat{Sent_t^{Inst}} + \varepsilon_t \qquad \textit{(Model 3.c)}$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + b_1 \widehat{Sent_t^{Indv}} + b_2 \widehat{Sent_t^{Inst}} + b_3 u_t^{Indv} + b_4 u_t^{Inst} + \varepsilon_t \qquad \textit{(Model 3.d)}$$
$$h_t = \omega + \theta \varepsilon_{t-1}^2 + \beta h_{t-1}$$

Model 1 is the benchmark where there is no sentiment parameter. Model 2 uses investor survey data directly, whereas in Model 3 I use fitted sentiment parameters along with their residuals. *Table 9* reports the estimation results with Robust Standard errors.

Table 9

GARCH-in-mean for Monthly Series (Estimation with Conditional Volatility)

	M1	M2a	M2b	M2c	M3a	M3b	M3c	M3d
μ_{t-1}	-0.0059	-0.0071	-0.0040	-0.0044	-0.0071	-0.0029	-0.0056	-0.0036
	(-1.593)	(-1.737)	(-0.938)	(-0.945)	(-1.620)	(-0.291)	(-0.229)	(-0.782)
r_{t-1}	-0.1070	0.0336	0.0344	0.0643	0.0130	0.0594	-0.3488	0.0876
	(-1.648)	(0.407)	(0.310)	(0.783)	(0.152)	(0.678)	(-4.161)	(0.851)
r_{t-2}	0.0652	-0.0201	0.2622	0.2069	0.0055	0.2884	0.0753	0.2374
	(0.757)	(-0.213)	(0.994)	(0.815)	(0.040)	(3.769)	(0.521)	(1.359)
h_t	0.1142	0.1431	-0.2226	-0.1147	0.1524	-0.6340	-0.2274	-0.2952
	(0.564)	(0.560)	(-0.429)	(-0.431)	(0.499)	(-0.240)	(-0.123)	(-0.347)
$Sent_t^{Indv}$		0.0561		0.0227				
		(4.481)		(1.831)				
$Sent_t^{Inst}$			0.2630	0.2387				
			(8.070)	(4.744)				
$\widehat{Sent_t^{Indv}}$					0.0738		0.0075	0.0130
					(2.490)		(0.264)	(0.829)
$\widehat{Sent_t^{Inst}}$						0.3149	0.2691	0.2997
						(10.554)	(8.067)	(8.305)
u_t^{Indv}					0.0509			0.0253
					(3.828)			(2.094)
u_t^{Inst}						0.2437		0.2119
						(9.049)		(4.917)
ω	0.0001	0.0001	0.0003	0.0002	0.0001	0.0005	0.0000	0.0002
	(1.398)	(1.060)	(0.957)	(1.067)	(1.120)	(0.339)	(0.151)	(0.790)
ε_{t-1}^2	0.1603	0.1208	0.1166	0.1041	0.1224	0.1896	0.1137	0.1183
	(1.943)	(2.577)	(1.481)	(1.684)	(2.753)	(0.315)	(1.915)	(1.242)
h_{t-1}	0.8174	0.8636	0.6899	0.7285	0.8662	0.6624	0.8843	0.7148
	(11.488)	(19.544)	(7.633)	(5.921)	(23.643)	(4.215)	(1233.1)	(6.849)
skew	0.5749	0.6081	0.9162	0.8748	0.5921	0.9462	0.0266	0.9382
	(6.591)	(4.949)	(5.003)	(3.525)	(5.476)	(9.913)	(0.348)	(6.179)
kurtosis	9.6469	5.3310	3.2957	3.5559	4.9474	2.5094	15.2224	3.0735
	(1.261)	(2.010)	(1.168)	(1.397)	(2.216)	(1.328)	(1.976)	(1.402)
LLH	210.25	222.67	246.22	248.83	223.00	247.82	227.10	251.00

In all models, the estimated mean of excess returns, which is conditional on past returns is negative and insignificant. Lag excess returns AR(1,2) are mostly positive but also are insignificant. The relationship between estimated excess returns and conditional volatility (h_t) is mostly negative and insignificant. According to Lee et al. (2002), this would indicate a contradiction to the CAPM's claim as investors are being penalized for the risk they take. The estimated GARCH and ARCH coefficients are mostly significant throughout each specification, which confirms the hypothesis that high volatility is followed by high and low volatility is followed by low volatility. The level of excess skewness is significant in almost all specifications. However, the models do a good job of dealing with the excess kurtosis, which is almost insignificant in all specifications. Contrary to Fisher and Statman (2000), I found a positive and statistically significant relationship between *changes* in sentiment *Bull Ratio* of investors and S&P 500 excess returns in the following month (M2a, M2b). The relationship is stronger for the *institutional* investors, indicating that the opinions of *institutional* investors matter more relative to the opinions of *individual* investors on S&P 500 excess returns, which is also confirmed when both are used together in the mean equation (M2c). Using each group of investors separately, I found that an increase of 1.0 percentage point in the *Bull Ratio* of *institutional* investors is associated, on average, with about 0.26 pp increase in S&P 500 excess returns in the following month. An increase of 1.0 percentage point in the *Bull Ratio* of *individual* investors is associated, on average, with about 0.05 pp increase in S&P 500 excess returns in the following month. The log-likelihood values are substantially improved when adding sentiment parameters. This suggests that these variables, when used as explanatory variables, increase the goodness of fit in GARCH models. Test results also indicate that *observable* and *unobservable* components of investor sentiment are important elements in explaining S&P 500 excess returns. They are all positive and significant, suggesting that the selected macro variables do a good job of explaining shifts in investors' *Bull Ratio*. For individual investors, *observable* and *unobservable* components together have a bigger impact on excess returns by reducing the insignificant effects of AR(1,2) on returns, compared to when it is used directly from the surveys (M2a vs. M3a). This impact is even bigger for institutional investors, which reduces the insignificant effect of conditional volatility (h_t) on returns (M2b vs. M3b). Again, when used together in the mean equation, the *observable* component of institutional sentiment has more significant impact on excess returns than that of individual investors (M2c vs. M3c). Lastly, the *unobservable* component of individual sentiment is stronger and more significant than the *observable* component (M2c vs. M3c vs. M3d).

A close inspection reveals that all models capture the high volatility and subsequent market crisis for the dot-com crisis and the housing crisis in *Figure 7* below. It confirms the hypothesis that the conditional volatility was low during the expansion period between 2003 and 2007, while it was high during both crises.

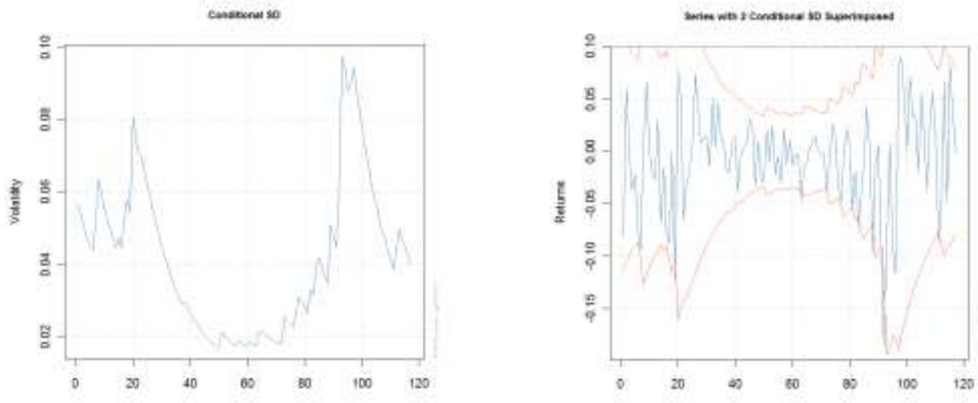


Figure 7

In *Table 10*, the performance of a GARCH specification is examined by computing the distributional statistics of standardized (and standardized squared) residuals for 20 lags along with an ARCH-LM test for 10 lags.[18] If the model fit the data properly, we expect those residuals to be free of serial correlation. All test statistics are insignificant at the 5% level, suggesting that the model succeeded in removing the serial correlations from the data series. I also report Akaike information criterion (AIC), which is asymptotically optimal in selecting the model with the least mean squared errors. The model with the minimum AIC value is usually preferred.

Table 10	M1	M2a	M2b	M2c	M3a	M3b	M3c	M3d
AIC	-3.41	-3.60	-4.00	-4.03	-3.59	-4.01	-3.66	-4.03
$Q(20)$	26.03	26.28	19.55	19.73	27.04	20.16	*45.33*	21.03
$Q^2(20)$	8.87	7.70	2.17	1.71	6.67	2.85	5.78	1.90
ARCH Lag[10]	5.36	4.53	0.88	0.58	3.31	1.36	2.55	0.78
Adjusted Pearson	24.37	49.80	38.96	40.66	43.02	44.05	48.10	*72.01*

***Note**: Italicized estimations are significant at 5%*

[18] The ARCH-LM test tests the return series for the null hypothesis is random (the coefficients are zero). Box-Pierce Q-statistic (similar to Ljung-Box test) for testing serial correlation in standardized residuals and squared standardized residuals for lags up to 20, $Q(20)$ and $Q^2(20)$, respectively.

V.4.2. *GARCH-in-mean for Monthly Series (with Implied Volatility Data)*

I repeat the analysis as in Section V.4.1 and estimate the following equations using the GARCH(1,1) model with the skewed Student's t-distribution, where instead of adding conditional variance in the mean equation, I add log changes in implied volatility (VIX) in the mean equation.

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + \varepsilon_t \qquad \textit{(Model V.1)}$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + bSent_t^{Indv} + \varepsilon_t \qquad \textit{(Model V.2.a)}$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + bSent_t^{Inst} + \varepsilon_t \qquad \textit{(Model V. 2.b)}$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + b_1 Sent_t^{Indv} + b_2 Sent_t^{Inst} + \varepsilon_t \qquad \textit{(Model V.2.c)}$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + b_1 \widehat{Sent_t^{Indv}} + b_2 u_t^{Indv} + \varepsilon_t \qquad \textit{(Model V.3.a)}$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + b_1 \widehat{Sent_t^{Inst}} + b_2 u_t^{Inst} + \varepsilon_t \qquad \textit{(Model V.3.b)}$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + b_1 \widehat{Sent_t^{Indv}} + b_2 \widehat{Sent_t^{Inst}} + \varepsilon_t \qquad \textit{(Model V.3.c)}$$
$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + b_1 \widehat{Sent_t^{Indv}} + b_2 \widehat{Sent_t^{Inst}} + b_3 u_t^{Indv} + b_4 u_t^{Inst} + \varepsilon_t \qquad \textit{(Model V.3.d)}$$
$$h_t = \omega + \theta \varepsilon_{t-1}^2 + \beta h_{t-1}$$

where $Vix_t = \ln\left(\frac{VIX_t}{VIX_{t-1}}\right)$. Because index price today represents the firms' future earnings and if *implied* volatility is desired to be used as a measure of risk, it should be more appropriate to add *implied* volatility into the mean equation rather than *past* volatility.

Table 11

GARCH-in-mean for Monthly Series (Estimation with Implied Volatility Data)

	MV1	MV2a	MV2b	MV2c	MV3a	MV3b	MV3c	MV3d
μ_{t-1}	-0.0055 (-1.577)	-0.0052 (-1.579)	-0.0041 (-1.065)	-0.0040 (-1.018)	-0.0056 (-1.731)	-0.0039 (-1.008)	-0.0051 (-0.223)	-0.0037 (-0.924)
r_{t-1}	-0.0630 (-0.228)	-0.0196 (-0.216)	0.0278 (0.232)	0.0521 (0.493)	0.0006 (0.085)	0.0348 (0.273)	-0.0772 (-0.045)	0.0666 (0.589)
r_{t-2}	0.1696 (1.884)	0.0870 (0.825)	0.1958 (1.211)	0.1827 (1.382)	0.0698 (0.655)	0.1994 (1.261)	0.1704 (0.514)	0.2000 (1.705)
$logVix_t$	-0.1312 (-5.927)	-0.1144 (-7.776)	-0.0782 (-4.432)	-0.0774 (-4.104)	-0.1202 (-9.532)	-0.0758 (-3.871)	-0.1173 (-0.697)	-0.0780 (-3.455)
$Sent_t^{Indv}$		0.0456 (3.247)		0.0206 (2.334)				
$Sent_t^{Inst}$			0.1872 (6.780)	0.1627 (3.962)				
$\widehat{Sent_t^{Indv}}$					0.0169 (0.661)		-0.0041 (-0.072)	-0.0026 (-0.146)
$\widehat{Sent_t^{Inst}}$						0.2088 (4.429)	0.1150 (2.118)	0.1996 (4.302)
u_t^{Indv}					0.0506 (4.036)			0.0260 (2.761)
u_t^{Inst}						0.1840 (7.293)		0.1517 (4.044)
ω	0.0001 (0.278)	0.0001 (1.234)	0.0002 (1.756)	0.0001 (1.383)	0.0001 (1.390)	0.0002 (1.588)	0.0001 (0.044)	0.0001 (1.371)
ε_{t-1}^2	0.2346 (0.520)	0.1544 (2.209)	0.1763 (1.587)	0.1555 (1.501)	0.1578 (2.423)	0.1854 (1.484)	0.2030 (0.129)	0.1729 (1.553)
h_{t-1}	0.7387 (1.628)	0.8076 (12.556)	0.6457 (6.881)	0.6970 (6.867)	0.8069 (14.867)	0.6379 (6.508)	0.7512 (0.283)	0.6730 (5.916)
skew	0.8706 (3.500)	0.7467 (4.516)	0.8539 (6.499)	0.8665 (7.436)	0.7557 (5.726)	0.8669 (6.505)	0.8663 (0.337)	0.9087 (8.256)
kurtosis	3.8120 (0.952)	4.6522 (2.162)	3.7873 (2.040)	3.6935 (2.039)	4.8306 (2.168)	3.5672 (1.979)	4.4480 (0.217)	3.8169 (1.836)
LLH	235.23	245.10	258.69	261.51	246.07	258.89	238.00	262.83

Consistent with my earlier claim, *Table 11* shows a negative and statistically significant relationship between *implied* volatility and monthly excess returns. The estimated coefficients have similar signs and significance, if not better, as in *Table 9*. The effects of both investors' sentiment on excess returns decreased when implied volatility was added into the model, which is more obvious for the *observable* component of institutional investors (M2b vs. MV2b and M3b vs. MV3b). All models have the similar skewness as before but with more significant kurtosis. Log-likelihood function has improved substantially throughout each specification.

Table 12	MV1	MV2a	MV2b	MV2c	MV3a	MV3b	MV3c	MV3d
AIC	-3.83	-3.98	-4.22	-4.25	-3.98	-4.20	-3.85	-4.23
$Q(20)$	17.64	18.87	24.26	22.47	19.02	24.36	19.90	22.84
$Q^2(20)$	10.11	5.59	2.52	1.47	7.71	2.85	10.60	1.82
ARCH Lag[10]	3.46	2.58	0.78	0.41	3.93	0.92	2.75	0.55
Adjusted Pearson	40.47	44.71	34.54	61.84	41.32	44.71	45.56	66.08

***Note**: Italicized estimations are significant at 5%*

Information criteria reported in *Table 12* is consistent with that of in *Table 10*. All test statistics are insignificant at the 5% level, suggesting that all models do a good job in removing the serial correlations from the data series.

V.4.3. *GJR-GARCH-in-mean and Implied Volatility Data for Weekly Series*

It is claimed that GARCH models are more appropriate for high frequency data series, *Figure 8*. In Section IV.1 and IV.2, I analyzed market excess returns with 120 monthly series. In this section, I repeat the same methodology for weekly series. There are total 634 observations.

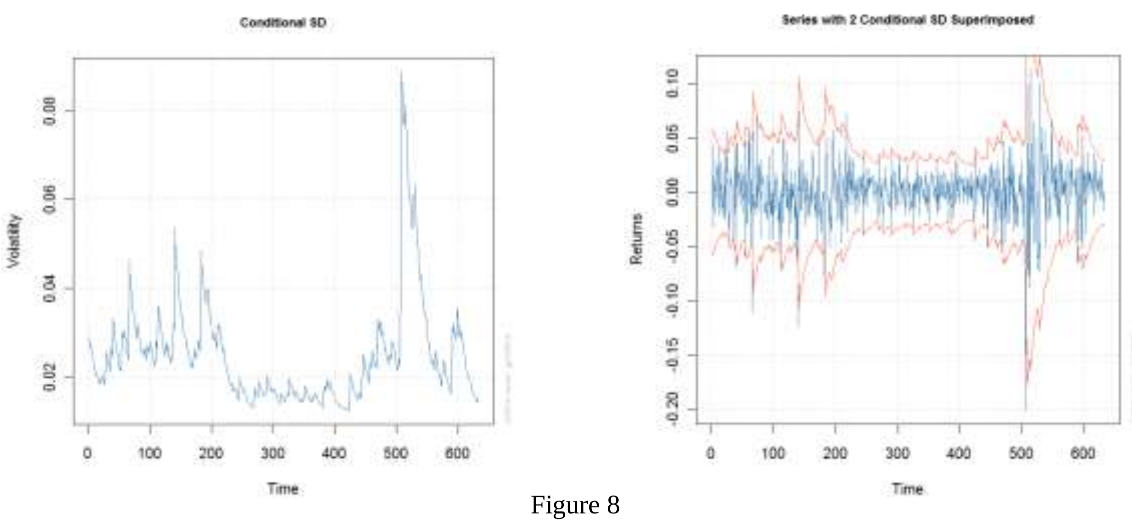


Figure 8

After testing various GARCH models, GJR-GARCH(1,1) with AR(2) seems to produce the best results. There are total eight models to estimate. I report the estimated coefficients in *Table 13*.

$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + \varepsilon_t$ *(Model W.1)*

$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + bSent_t^{Indv} + \varepsilon_t$ *(Model W.2.a)*

$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + bSent_t^{Inst} + \varepsilon_t$ *(Model W.2.b)*

$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta h_t + b_1 Sent_t^{Indv} + b_2 Sent_t^{Inst} + \varepsilon_t$ *(Model W.2.c)*

$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + \varepsilon_t$ *(Model W.V.3.a)*

$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + bSent_t^{Indv} + \varepsilon_t$ *(Model W.V.3.b)*

$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + bSent_t^{Inst} + \varepsilon_t$ *(Model W.V.3.c)*

$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta Vix_t + b_1 Sent_t^{Indv} + b_2 Sent_t^{Inst} + \varepsilon_t$ *(Model W.V.3.d)*

$h_t = \omega + \theta \varepsilon_{t-1}^2 + \beta h_{t-1}$

First, coefficients of conditional volatility are negative and insignificant throughout each specification, whereas coefficients of implied volatility are negative and significant. Consistent with monthly series, the estimated sentiment coefficients are positive and significant. Second, the impact of changes in individual investor sentiment on weekly log returns declines when implied volatility, rather than conditional volatility, is added into the mean equation, while the impact of changes in institutional investor sentiment on weekly log returns remains relatively stronger in each model. Finally, when implied volatility is used, the value of log-likelihood function is improved substantially.

Table 13

GJR-GARCH-in-mean and Implied Volatility Data for Weekly Series

	MW1	MW2a	MW2b	MW2c	MWV3a	MWV3b	MWV3c	MWV3d
μ_{t-1}	0.0009	0.0011	0.0011	0.0012	0.0010	0.0010	0.0012	0.0012
	(1.318)	(1.849)	(2.056)	(2.373)	(1.161)	(1.180)	(0.988)	(1.055)
r_{t-1}	-0.1362	-0.2311	-0.2599	-0.3012	-0.0292	-0.0560	-0.1095	-0.1166
	(-3.158)	(-5.045)	(-5.623)	(-6.321)	(-0.626)	(-1.138)	(-2.143)	(-2.213)
r_{t-2}	0.0042	0.0034	-0.0583	-0.0548	0.0205	0.0330	0.0048	0.0135
	(0.212)	(0.259)	(-1.347)	(-1.242)	(0.597)	(0.797)	(0.361)	(0.362)
h_{t}	-0.3452	-0.4871	-0.4538	-0.5092				
	(-0.440)	(-0.827)	(-0.720)	(-0.920)				
$logVix_t$					-0.1235	-0.1230	-0.1182	-0.1177
					(-13.518)	(-13.438)	(-13.969)	(-13.999)
$Sent_t^{Indv}$		0.0220		0.0146		0.0069		0.0040
		(4.669)		(3.223)		(2.503)		(1.600)
$Sent_t^{Inst}$			0.1649	0.1507			0.0824	0.0795
			(9.695)	(8.921)			(7.914)	(7.631)
ω	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(7.026)	(5.082)	(6.291)	(12.071)	(0.736)	(0.662)	(0.286)	(0.289)
ε_{t-1}^2	0.0000	0.0000	0.0000	0.0000	0.0563	0.0495	0.0558	0.0534
	(0.000)	(0.000)	(0.000)	(0.000)	(0.593)	(0.511)	(0.289)	(0.287)
$I_{t-1}\varepsilon_{t-1}^2$	0.1319	0.1217	0.1487	0.1441	0.1553	0.1443	0.1139	0.1144
	(3.755)	(3.516)	(3.337)	(3.283)	(1.816)	(1.697)	(0.814)	(0.818)
$h_{\mathrm{t-1}}$	0.9079	0.9137	0.8910	0.8950	0.8407	0.8533	0.8671	0.8695
	(63.958)	(74.219)	(41.211)	(48.807)	(31.007)	(34.862)	(35.193)	(35.013)
skew	0.7608	0.7320	0.8640	0.8450	0.9600	0.9623	1.0121	1.0149
	(13.837)	(11.911)	(16.123)	(16.111)	(12.230)	(12.080)	(8.605)	(9.205)
shape	8.9631	9.8864	7.8908	8.4624	6.3740	6.3302	5.3676	5.4142
	(3.513)	(3.060)	(3.129)	(2.877)	(3.349)	(3.580)	(2.869)	(3.020)
LLH	1503.11	1515.01	1547.35	1552.87	1756.92	1760.41	1783.94	1785.19

Table 14 confirms that all test statistics are insignificant at 5% level and all models do a good job in removing the serial correlations from weekly series.

Table 14	MV1	MV2a	MV2b	MV2c	MV3a	MV3b	MV3c	MV3d
AIC	-4.73	-4.76	-4.86	-4.88	-5.53	-5.54	-5.61	-5.61
$Q(20)$	13.07	16.97	20.52	21.04	25.79	26.60	28.88	29.14
$Q^2(20)$	20.72	20.36	9.32	9.92	15.72	15.98	11.60	11.52
ARCH Lag[10]	13.38	12.82	6.57	6.90	11.90	11.93	9.99	9.80
Adjusted Pearson	50.91	33.82	49.01	52.18	4.97	59.30	41.89	45.06

***Note**: Italicized estimations are significant at 5%*

VI. Chapter Conclusion

I used various GARCH models with different combination of external variables in the mean equation in order to build a confidence bound around the estimated coefficients. There were fewer observations for monthly data, which is a disadvantage for GARCH models. I include weekly series in the last section to assure that the results produced from monthly series were consistent with a high frequency data and it can be carried on to daily series in Chapter III. We observe that the relationship between index returns and conditional volatility (h_t) remains insignificant and inconsistent within the groups and across the GARCH models with and without the asymmetric effect. Instead of conditional volatility, when *implied* volatility (VIX) is used as a measure of risk, the goodness of fit and overall test statistics have been improved substantially. Also adding external variables in the conditional variance such as risk free rate or using GJR-GARCH models with AR(1) did not change the results. In this chapter, I developed a methodology that not only measures the effects of *explicit* (observable) sentiment but also the effects of *implicit* (unobservable) sentiment. I find a positive and statistically significant relationship between *changes* in the sentiment *Bull Ratio* of both *institutional* and *individual* investors and the S&P 500 excess returns in the following month. Overall, we can argue the fact that individual investors usually follow what institutional investors do; however, test statistics indicate that both direct and observable *individual* investor sentiment have less effects on S&P 500 excess returns. There might be a few reasons for this: One, they may not be following through their opinions when they actually trade. Two, the dollar value of their investments invested in stock market is relatively smaller than the total value invested by the institutional investors. Three, there are other economic and market factors (which are not used here) which can do a better job of explaining changes in observable *individual* investor sentiment. Nevertheless, the selected macro variables explaining the changes in both investor sentiment overall do a good job.

Chapter III

Effects of Investor Sentiment and Implied Volatility on Daily Returns

I. Introduction

In this section, I carry out the same methodology adopted in Chapter II for daily series. Earlier studies analyzing high frequency series use ARCH in order to capture the time series properties (e.g., serial correlation) and to forecast underlying return volatility.[19] However, those models are often criticized because they gave little evidence on the economic forces behind the volatility. A common approach for comparing different time series models is to ask which model fits the data best. There is a wide acceptance of GARCH models when modeling daily stock returns because: First, there is evidence that these models fit nonlinear return series better. Second, the parameter estimates of GARCH models are usually statistically significant. In Chapter I, I argue that there is a negative significant relationship between index returns and implied volatility. In Chapter II while quantifying the effects of investor sentiment on index returns, I show that when implied volatility is used in the mean equation, it does a better job of explaining returns compared to conditional volatility. We can argue that there can be many distinctly different reasons why the current value of a time series can depend nonlinearly on its own past. However, that is not the objective here. Instead, in this chapter, combining my findings in Chapter I and Chapter II, I investigate how current value of daily returns depends on changes in investor sentiment and how this relationship differs under different implied (not past) volatility states, especially when measuring daily mean reversion in returns.

[19] See Diebold (1986), Stock (1987, 1988), and Lamoureux and Lastrapes (1990).

II. Methodology

II.1. *Benchmark*

In benchmark, I apply a simple GARCH methodology to determine the changes in daily returns, which is similar to the one used in Chapter II. There are no exogenous regressors either in the mean equation or in the variance equation. A simple $AR(2) - GARCH(1,1)$ model is given by:

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \varepsilon_t \ ;$$

$$\varepsilon_t \sim (0, h_t) \ \ where\ \varepsilon_t = z_t \sqrt{h_t}\ and\ z_t \sim (0,1)\ ;$$

$$h_t = \omega + \theta \varepsilon_{t-1}^2 + \beta h_{t-1}\ ;$$

where
r_t: Daily S&P 500 log returns for period *t*: $r_t = \ln(P_t) - \ln(P_{t-1})$
μ_{t-1}: Mean of r_t conditional on past information
h_t: Variance
ε_t: Residual

II.2. *Daily Returns and Measure of Investor Sentiment as a Reference Point*

In behavioral finance, anchoring is a decision-making process under uncertainty, and usually starts with a certain reference point and then adjusts it insufficiently to reach a final conclusion. Depending on the accuracy of the reference point, the final conclusion may vary. Fuller (1998) defines saliency as a situation in which events occur infrequently but people tend to overestimate the probability of such an event occurring in the future if they have recently observed such an event. He gives commercial airline crashes as an example, where if an airplane crash is reported in the media, people will overestimate the probability of a crash occurring in the near future. Kahneman (1992) claims that most (investment) decisions involve multiple reference levels, which can be used to separate the time series data into the regions of desirable and undesirable outcomes. In finance, people constantly update their beliefs as they receive financial data. Investors put more weight on recent behaviors of stock market movements; usually the most recent (past returns) information can be viewed as an *anchor*. We can observe (and measure) how investors revise their expectations using survey data, where they state their opinions about the future direction of the market. In Chapter II, I found a positive significant relationship between *changes* in sentiment *Bull Ratio* of both *institutional* and *individual* investors and the S&P 500 excess in the next period. In this section, I use those estimation results (fitted values and residuals) to measure the expected market returns for the following period. I have two methodologies that are adopted for monthly and weekly series, separately.

Long-term return studies in behavioral models claim that *representative* judgment bias can create overconfidence. When a company has a consistent history of earnings growth over several years, investors might conclude that the past history is representative of underlying earning potential. It is defined as the "law of small numbers": people underweight long-term averages, putting more weight on the recent trend, and less weight on prior trends. Shefrin (2002) believes that representative bias is the reason why investors expect high returns from safe (less volatile) stocks. Under uncertainty, investors are systematically overconfident in their ability to succeed or their knowledge to forecast stock returns in particular when they think of themselves as experts. Inspired by Barberis et al. (1998), who offer an explanation for under or overreaction based on a *learning model* in which actual earnings follow a random walk, but individuals believe that earnings either follow a *steady growth trend* or are *mean reverting*, I introduce two methodologies, one of which uses *monthly* macroeconomic data as a trend indicator when measuring investor sentiment. Particularly, I use both *observable* and *unobservable* components of investor sentiment that are first regressed on the selected macroeconomic variables along with their residuals.[20] My second methodology uses *weekly* survey data directly. Particularly, both *individual* and *institutional* sentiment *Bull Ratios* regressed on market returns.

$$E(R_T) = \sum_{i=1}^{2} \hat{\alpha}_i r_{T-i} + \hat{\beta}_1 \widehat{Sent_t^{Indv}} + \hat{\beta}_2 \widehat{Sent_t^{Inst}} + \hat{\beta}_3 u_t^{Indv} + \hat{\beta}_4 u_t^{Inst} \qquad \textit{(Monthly Series)}$$

$$E(R_T) = \sum_{i=1}^{2} \hat{\alpha}_i r_{T-i} + \hat{\delta} logVix_t + \hat{\beta}_1 Sent_t^{Indv} + \hat{\beta}_2 Sent_t^{Inst} \qquad \textit{(Weekly Series)}$$

---

[20] See Table 1 in Chapter II for the selected macro economic variables that are announced monthly. Instead of taking survey data directly, I use the effects of macroeconomic variables on investor sentiment first and then I measure the expected market returns by taking the fitted values estimated from Model 3 in Chapter II, which had the highest LLH value among other models.

where $E(R_T)$ is the fitted values from the regression; $\widehat{Sent_t^{Indv}}\ and\ \widehat{Sent_t^{Inst}}$ are (*observable*) changes in the sentiment *Bull Ratio* of *individual* and *institutional* investors, which are obtained after they are regressed on macroeconomic variables; $u_t^{Indv}\ and\ u_t^{Inst}$ are the residuals (*unobservable* changes in the sentiment *Bull Ratio*); $Sent_t^{Indv}$ and $Sent_t^{Inst}$ are changes in the sentiment *Bull Ratio* of *individual* and *institutional* investors; and R_T is the *realized* S&P 500 excess returns at the *end* of each time period *T*. Recall from Section V.1 in Chapter II, in order to convert weekly survey series into a monthly data, I use the percentage of sentiment *Bull Ratio* of both investors in the *last week* of each month as a measure of investor sentiment. The implicit assumption made here is that the *realized* market return, R_T plays a role of an *anchor* when compared to $E(R_T)$. I define gamma (Γ_T) representing the difference between *expected* and *realized* returns for any given time T:

$$\Gamma_\mathrm{T} = E(R_T) - R_T \ ;$$

such that when gamma is positive, it is interpreted as investors being confident, *bullish* sentiment and when gamma is negative, it is interpreted as investors being pessimistic, *bearish* sentiment.

$\Gamma_\mathrm{T} \geq 0$:	*Bullish* effect,	if $E(R_T) \geq R_T$
$\Gamma_\mathrm{T} < 0$:	*Bearish* effect,	if $E(R_T) < R_T$

For *monthly* series, Gamma (Γ_T) measures the amount that will be reflected on the next month's daily returns as a *bullish* or *bearish* effect. This amount will be distributed on daily returns over the next month. However, the distribution will be weighted. Given that investors would want to revise their expectations without any delay, the effect of revision is likely to be stronger during the first week of the month and to die out gradually as the month continues. For any given amount of gamma, I use the following weights, ∂_k.[21] For weekly series, I only use the reported surveys directly without regressing them on selected macro economic variables. Therefore, there is no need to convert the data into a monthly series, instead we can just use them as given in each week. This means that we do not need to make any adjustments when distributing the effects of

[21] The fifth week is added if there is more than 20-business days in a month. Also, this weighting regime is not an absolute criteria. There can be other regimes which represent the distribution better. This particular one is an illustration and is adopted for a convenience. When we use weekly series, we will not need a weighting regime.

investor sentiment in the dataset i.e. $\partial_k = 1$ in each week. The following chart summarizes the weighting regime that is used for *monthly* series for any given week *k* in a month *T*:

for each week k in month T	
$k = 1$	$\partial_1 = 0.5$
$k = 2$	$\partial_2 = 0.25$
$k = 3$	$\partial_3 = 0.125$
$k = 4$	$\partial_4 = 0.0625$
$k = 5$	$\partial_5 = 0.0625$
	Total= 1.00

such that

for each week *k* If $\Gamma_{T-1} \neq 0$	$\partial_1 * \Gamma_{T-1}$ $0.5 * \Gamma_{T-1}$ first week	$\partial_2 * \Gamma_{T-1}$ $0.25 * \Gamma_{T-1}$ second week	$\partial_3 * \Gamma_{T-1}$ $0.125 * \Gamma_{T-1}$ third week	$\partial_4 * \Gamma_{T-1}$ $0.0625 * \Gamma_{T-1}$ fourth week	$\partial_5 * \Gamma_{T-1}$ $0.0625 * \Gamma_{T-1}$ fifth week	Γ_T
month T-1	*month T*					*month T+1*

In a contrarian strategy, some investors believe that widespread optimism (pessimism) about market conditions can result in high (low) valuations that will eventually lead to drops (spikes), when those expectations do not happen. The measure of gamma here is intended to show investors' widespread optimism and pessimism. This measure is believed to have effects on the degree of mean-reverting behavior of stock returns. Earlier studies showed that the estimated autocorrelations of short-term returns were close to zero, which provided support for the random walk hypothesis. In the debate about stock market efficiency, Summer (1986), Poterba and Summer (1988), and Fama and French (1988a) have shown long-term *temporary* deviations of stock price from its fundamental value resulting in mean-reverting behavior of stock prices. Choe, Nam, and Vahid (2007) argue that the sign of the first-order return autocorrelation for the mean reversion should depend on the lag structure of the transitory components of underlying stock prices. In Chapter II, using the AR(2)-GARCH-in-mean process, I found that the sign of the first-order return had mixed results associated with insignificant coefficients for *monthly* series. However, when AR(2)-GJR-GARCH process with implied volatility in the mean equation was used, the sign of the first-order return was negative and significant for *weekly* series. In this chapter, using GARCH(1,1) process, I first find the sign of the AR(1) process in daily returns to be negative and significant. I then investigate the effect of investor sentiment on daily mean reversion of index returns by adding the value of gamma that is multiplied by the previous day's return. If investors are bullish (positive gamma), I expect the sign of "*bullish effect*" to be positive, which reduces the degree of mean reversion, whereas if investors are bearish (negative gamma), I expect the sign of "*bearish effect*" to be negative, which increases the degree of mean reversion on daily index returns. The economic reasoning behind this relationship is that when investors are optimistic (pessimistic) about the market, there is less (more) expected-implied volatility in stock prices which suggests that forthcoming reversion in prices will be small (large). The *bullish* and *bearish* effects on day *t* for any given week k in month *T* is given by:

Bullish effect	$(\partial_k * 100 * \Gamma_{\mathrm{T}})\mathrm{d}_{\mathrm{T}}\mathrm{r}_{\mathrm{t-1}}$
Bearish effect	$(\partial_k * 100 * \Gamma_{\mathrm{T}})(1-\mathrm{d}_{\mathrm{T}})\mathrm{r}_{\mathrm{t-1}}$

$$d_T = \begin{cases} 1, & if \;\; \Gamma_T \geq 0 \\ 0, & otherwise \end{cases} \qquad \text{and} \qquad (1-d_T) = \begin{cases} 1, & if \;\; \Gamma_T < 0 \\ 0, & otherwise \end{cases}$$

where $\partial_k = 1$ for *weekly* series.

The mean and the variance equations for monthly and weekly series are given by:

bullish effect　　　　bearish effect

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \alpha_3(\partial_k * 100 * \Gamma_{\mathrm{T-1}})\mathrm{d}_{\mathrm{T-1}}\mathrm{r}_{\mathrm{t-1}} + \alpha_4(\partial_k * 100 * \Gamma_{\mathrm{T-1}})(1-\mathrm{d}_{\mathrm{T-1}})\mathrm{r}_{\mathrm{t-1}} + \varepsilon_t;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \alpha_3(100 * \Gamma_{\mathrm{T-1}})\mathrm{d}_{\mathrm{T-1}}\mathrm{r}_{\mathrm{t-1}} + \alpha_4(100 * \Gamma_{\mathrm{T-1}})(1-\mathrm{d}_{\mathrm{T-1}})\mathrm{r}_{\mathrm{t-1}} + \varepsilon_t;$$

or in short:

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \alpha_3 Bullish\ Effect_t + \alpha_4 Bearish\ Effect_t + \varepsilon_t;$$

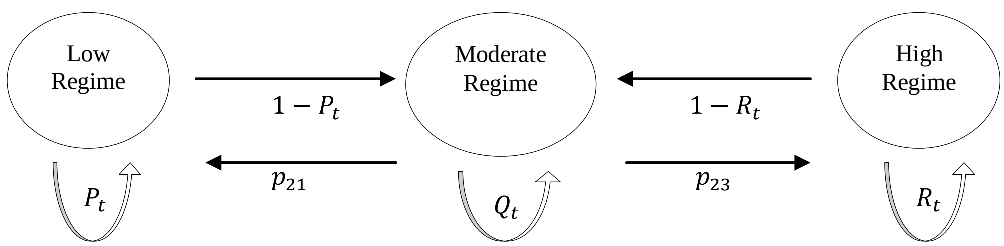
where $\varepsilon_t = \sqrt{h_t} z_t$; $\varepsilon_t \sim (0, h_t)$; $z_t \sim iidN(0,1)$; and with the same conditional variance, $h_t = b_0 + b_1 \varepsilon_{t-1}^2 + h_{t-1}$; α_i of $\mathrm{r}_{\mathrm{t-i}}$ for $i = 1$ captures *mean reversion* in daily returns and the sign of α_1 is expected to be negative. As claimed, I argue that the degree of mean reversion depends on the value of gamma: On average, I expect the sign of the coefficient for *bullish* sentiment (α_3) to be positive, which reduces the negative effect of α_1 on daily returns. On the other hand, I expect the sign of the coefficient for *bearish* sentiment (α_4) to be negative, which increases the negative effect of α_1 on daily returns.

II.3. *Daily Returns, Investor Sentiment, and Implied Volatility*

II.3.1. *Review (Regime Switching, RS-GARCH Models)*

II.3.1.1. *Conventional Stochastic RS-GARCH Models*

A Regime Switching GARCH (RS-GARCH) model was introduced by Hamilton (1989), where conditional variance of the model depends on past states. Gray (1995 and 1996) improves RS-GARCH models by introducing a path-independent GARCH process, where each conditional variance (of *stock returns*) depends only on the current regime, not on the entire past history of the process. For a three-state process, a transition diagram is given by:



where P_t, Q_t, and R_t are the probabilities of staying at the same regimes: regime 1, regime 2, and regime 3, respectively. The transition matrix with three-state Markov process is given by:

$$P = \begin{bmatrix} p(1,1) & p(1,2) & p(1,3) \\ p(2,1) & p(2,2) & p(2,3) \\ p(3,1) & p(3,2) & p(3,3) \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

For a three-state Markov process $S = \{1,2,3\}$, assume that s_{t+1} follows a one-order Markov process, which can be described by transition probabilities:

$$Prob(s_{t+1} = j | s_t = i, s_{t-1} = k, \dots . y_t, y_{t-1}, \dots .) = Prob(s_{t+1} = j | s_t = i) = p(i,j) = p_{ij};$$

with $(i,j) \epsilon SxS$. Each element in a transition matrix, p_{ij}, is the probability that state i is followed by state j. For example, p_{12} represents the regime switching probability of being in regime 2 at time *t* given that the market was in regime 1 at time *t-1*. Furthermore, the sum of the each elements in each and every row is equal to one, $\sum_{j \epsilon S}^{3} p(i,j) = 1$. Using a GARCH(1,1) model where each conditional variance (of *stock returns*) depends only on the current regime, we can define the conditional mean and the conditional variance as

$$r_t = \mu\left[\theta_\mu(S_t), \emptyset_{t-1}\right] + \sqrt{h[\theta_h(S_t), \emptyset_{t-1}]}z_t$$
$$h[\theta_h(S_t), \emptyset_{t-1}] = h_{t,i} = b_{i,0} + b_{i,1}\varepsilon_{t-1}^2 + b_{i,2}h_{t-1}$$

where $r_t = \ln\left({}^{P_t}/_{P_{t-1}}\right)$; $\varepsilon_{t,i} = \sqrt{h_{t,i}}z_t$ with $\varepsilon_{t,i} \sim N(0, h_{t,i})$; and $z_t \sim iidN(0,1)$. $\emptyset_{t-1}$ is the available information set up to time *t-1*. $\theta_\mu(S_t) = \{\alpha_{i,0}, \alpha_{i,1}\}$ if $\mu\left[\theta_\mu(S_t), \emptyset_{t-1}\right] = \alpha_{i,0} + \alpha_{i,1}\mathrm{r}_{\mathrm{t}-1}$ and $\theta_h(S_t) = \{b_{0i}, b_{1i}, b_{2i}\}$ are state-dependent vectors of unknown parameters. S_t is the *unobserved* regime at time *t*, where $S = \{1,2,3\}$ for each state $i \epsilon S$. If the distribution of r_t is assumed to be normal (although it can also be a mixture of other distributions), the model can be generalized to allow not only the parameters but also the functional forms to vary over time. For each regime i, $r_t | \emptyset_{t-1} = \Phi(c_i + d_i \mathrm{r}_{\mathrm{t}-1})$ with probabilities $p_{t,i}$, where probability distributions over three-state for any given time *t* are: $Pr(S_t = 1 | \emptyset_{t-1}) = p_{t,1}$, $Pr(S_t = 2 | \emptyset_{t-1}) = p_{t,2}$, and $Pr(S_t = 3 | \emptyset_{t-1}) = p_{t,3}$, where conditional *normality* in each regime, $N(\mu_{t,i}, h_{t,i}) = \Phi(c_i + d_i \mathrm{r}_{\mathrm{t}-1})$ represents the cumulative normal distribution function for $i = \{1,2,3\}$:

$$N(\mu_{t,i}, h_{t,i}) = f(r_t | S_t = i, \emptyset_{t-1}) = f(r_t | \emptyset_{t-1}) = \sum_{i=1}^{3} f(r_t, S_t = i | \emptyset_{t-1})$$

$$f(r_t | S_t = i, \emptyset_{t-1}) = \frac{1}{\sqrt{2\pi h_{t,i}}} \exp\left\{\frac{-(r_t - \mu_{t,i})^2}{2h_{t,i}}\right\}$$

Using the Gray (1996) path independent aggregation technique, we can write the conditional mean and conditional variance at time *t* as

$$\mu_t = E[r_t|\emptyset_{t-1}] = p_{t,1}\mu_{t,1} + p_{t,2}\mu_{t,2} + p_{t,3}\mu_{t,3}$$

$$h_t = E[r_t^2|\emptyset_{t-1}] - E[r_t|\emptyset_{t-1}]^2$$
$$= p_{t,1}\left(\mu_{t,1}^2 + h_{t,1}\right) + p_{t,2}\left(\mu_{t,2}^2 + h_{t,2}\right) + p_{t,3}\left(\mu_{t,3}^2 + h_{t,3}\right) - [p_{t,1}\mu_{t,1} + p_{t,2}\mu_{t,2} + p_{t,3}\mu_{t,3}]^2$$

where each μ_{t-i} and h_{t-i} are given earlier for each three-state. Finally, the log-likelihood function of *normally* distributed residuals z_t for some parameters ($\boldsymbol{\theta}$) of the model can be stated as:

$$L(\boldsymbol{\theta}) = \sum_{t=1}^{T} log\left[p_{t,1}\frac{1}{\sqrt{2\pi h_{t,1}}}\exp\left\{\frac{-\left(r_t - \mu_{t,1}\right)^2}{2h_{t,1}}\right\} + p_{t,2}\frac{1}{\sqrt{2\pi h_{t,2}}}\exp\left\{\frac{-\left(r_t - \mu_{t,2}\right)^2}{2h_{t,2}}\right\}\right.$$
$$\left. + p_{t,3}\frac{1}{\sqrt{2\pi h_{t,3}}}\exp\left\{\frac{-(r_t - \mu_{t,3})^2}{2h_{t,3}}\right\}\right]$$

where $p_{t,1} = Pr(S_t = 1|\emptyset_{t-1})$; $p_{t,2} = Pr(S_t = 2|\emptyset_{t-1})$; and $p_{t,3} = Pr(S_t = 3|\emptyset_{t-1})$ with

- $p_{t,1} = P_t * \frac{g_{t-1,1}\, p_{t-1,1}}{\sum_{i=1}^{3} g_{t-1,i}\, p_{t-1,i}} + p_{21,t} * \frac{g_{t-1,2}\, p_{t-1,2}}{\sum_{i=1}^{3} g_{t-1,i}\, p_{t-1,i}} + p_{31,t} * \frac{g_{t-1,3}\, p_{t-1,3}}{\sum_{i=1}^{3} g_{t-1,i}\, p_{t-1,i}}$
- $p_{t,2} = (1 - P_t) * \frac{g_{t-1,1}\, p_{t-1,1}}{\sum_{i=1}^{3} g_{t-1,i}\, p_{t-1,i}} + Q_t * \frac{g_{t-1,2}\, p_{t-1,2}}{\sum_{i=1}^{3} g_{t-1,i}\, p_{t-1,i}} + (1 - R_t) * \frac{g_{t-1,3}\, p_{t-1,3}}{\sum_{i=1}^{3} g_{t-1,i}\, p_{t-1,i}}$
- $p_{t,3} = \left(1 - p_{t,1} - p_{t,2}\right)$
- $g_{t-1,i} = f(r_{t-1}|S_{t-1} = i) = \frac{1}{\sqrt{2\pi h_{t-1,i}}}\exp\left\{\frac{-(r_t - \mu_{t-1,i})^2}{2h_{t-1,i}}\right\}$

We can transform z_t and can write the log-likelihood function above for the skewed Student-t distribution as well.

II.3.1.2. *Stochastic RS-GARCH where Implied Volatility determines Regimes*

In conventional regime switching (RS) GARCH models, the conditional regime probabilities are defined on the distribution of the variable (stock returns r_t) that is being explained by the model. In Chapter II, I argue that the conditional variance of returns (when used as a measure of time-varying risk) does a poor job compared to the implied volatility. In this section, I propose a regime switching model where regimes are defined not within the model but rather outside of the model particularly by *implied volatility*. According to the first-order Markov process in conventional RS-GARCH models, all available information set ($\emptyset_{t-1}$) up to the current state is defined on the distribution of past returns, which is given by $\emptyset_{t-1} = \tilde{r}_{t-1} = E(\mathrm{r}_{t-1}|\mathrm{r}_{t-2}, \mathrm{r}_{t-3}, \ldots) = \{\mathrm{r}_{t-1}, \mathrm{r}_{t-2} \ldots\}$. If we replace $\tilde{r}_{t-1}$ with $\widetilde{Vix}_{t-1}$, where $\widetilde{Vix}_{t-1} = \{Vix_{t-1}, Vix_{t-2} \ldots\}$, we would let the current state be defined on the distribution of *implied* volatility such that the information set up to the current state is written as martingale $\widetilde{Vix}_{t-1} = Vix_{t-1}$. Under normality, $Vix_t|\emptyset_{t-1} = N(\mu_{t,i}^{Vix}, (\sigma_{t,i}^{Vix})^2)$ and its cumulative normal distribution function in each state is given by:

$$f(Vix_t|S_t = i, \emptyset_{t-1}) = \frac{1}{\sqrt{2\pi(\sigma_{t,i}^{Vix})^2}} \exp\left\{\frac{-(Vix_t - \mu_{t,i}^{Vix})^2}{2(\sigma_{t,i}^{Vix})^2}\right\}$$

Further, we can assume that mean and variance in each period are still *stochastic* and conditional on the information set which depends on the current *implied* volatility level while keeping the idea of the Gray (1996) path-independent *aggregation* technique and writing the conditional mean and conditional variance at time *t* as:

$$\mu_t = E[r_t|\emptyset_{t-1}] = p_{t,1}\mu_{t,1} + p_{t,2}\mu_{t,2} + p_{t,3}\mu_{t,3}$$

$$h_t = E[r_t^2|\emptyset_{t-1}] - E[r_t|\emptyset_{t-1}]^2$$
$$= p_{t,1}(\mu_{t,1}^2 + h_{t,1}) + p_{t,2}(\mu_{t,2}^2 + h_{t,2}) + p_{t,3}(\mu_{t,3}^2 + h_{t,3}) - [p_{t,1}\mu_{t,1} + p_{t,2}\mu_{t,2} + p_{t,3}\mu_{t,3}]^2$$

which still have the same probability specifications:

- $p_{t,1} = P_t * \frac{g_{t-1,1}\, p_{t-1,1}}{\sum_{i=1}^{3} g_{t-1,i}\, p_{t-1,i}} + p_{21,t} * \frac{g_{t-1,2}\, p_{t-1,2}}{\sum_{i=1}^{3} g_{t-1,i}\, p_{t-1,i}} + p_{31,t} * \frac{g_{t-1,3}\, p_{t-1,3}}{\sum_{i=1}^{3} g_{t-1,i}\, p_{t-1,i}}$
- $p_{t,2} = (1 - P_t) * \frac{g_{t-1,1}\, p_{t-1,1}}{\sum_{i=1}^{3} g_{t-1,i}\, p_{t-1,i}} + Q_t * \frac{g_{t-1,2}\, p_{t-1,2}}{\sum_{i=1}^{3} g_{t-1,i}\, p_{t-1,i}} + (1 - R_t) * \frac{g_{t-1,3}\, p_{t-1,3}}{\sum_{i=1}^{3} g_{t-1,i}\, p_{t-1,i}}$
- $p_{t,3} = (1 - p_{t,1} - p_{t,2})$

but with a *different* cumulative normal distribution function, which is written over the distribution of *implied* volatility in each period:

$$g_{t-1,i} = f(Vix_t | S_t = i, \emptyset_{t-1}) = \frac{1}{\sqrt{2\pi(\sigma_{t,i}^{Vix})^2}} \exp\left\{\frac{-(Vix_t - \mu_{t,i}^{Vix})^2}{2(\sigma_{t,i}^{Vix})^2}\right\}$$

or we can assume a *non-stochastic* mean and variance such that

$$\mu[\theta_\mu(S_t), \emptyset_{t-1}] = \mu_t;$$

$$h[\theta_h(S_t), \emptyset_{t-1}] = h_t;$$

and the conditional mean and variance can be written as

$$r_t = \mu_{t-1} + \delta Vix[\theta_{Vix}(S_t), \emptyset_{t-1}] + \sqrt{h_t} z_t$$

$$h_t = \omega + \theta\varepsilon_{t-1}^2 + \beta h_{t-1}$$

where $\varepsilon_t = \sqrt{h_t} z_t$; $\theta_{Vix}(S_t) = Vix_t | \emptyset_{t-1}$; and $\mu_{t-1} = \mu + \alpha_0 \mathrm{r}_{\mathrm{t-1}} + \alpha_1 \mathrm{r}_{\mathrm{t-2}}$; and $Vix_t | \emptyset_{t-1}$ can be a change in level of VIX or log-change of VIX.

II.3.2. *Investor Sentiment and non-Stochastic RS-GARCH under Implied Volatility*

In benchmark, we have no regimes, where a simple non-stochastic GARCH(1,1) mean equation with implied volatility and investor sentiment is given by three equations, where the second and the third equations are adjusted for *monthly* and *weekly* series:

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta logVix_t + \varepsilon_t;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta logVix_t + \alpha_3(\partial_k * 100 * \Gamma_{\mathrm{T-1}})\mathrm{d}_{\mathrm{T-1}}\mathrm{r}_{\mathrm{t-1}} + \alpha_4(\partial_k * 100 * \Gamma_{\mathrm{T-1}})(1 - \mathrm{d}_{\mathrm{T-1}})\mathrm{r}_{\mathrm{t-1}} + \varepsilon_t;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \delta logVix_t + \alpha_3(100 * \Gamma_{\mathrm{T-1}})\mathrm{d}_{\mathrm{T-1}}\mathrm{r}_{\mathrm{t-1}} + \alpha_4(100 * \Gamma_{\mathrm{T-1}})(1 - \mathrm{d}_{\mathrm{T-1}})\mathrm{r}_{\mathrm{t-1}} + \varepsilon_t;$$

$h_t = \omega + \theta\varepsilon_{t-1}^2 + \beta h_{t-1}$.

DeBondt amd Thaler (1985) give psychological reasons why investors are subject to waves of optimisim and pessimism when they communicate regularly and act as a group, a *herding* bias. The efect of herding bias can be significant when the market is in a different volatility state. So I define a three-regime distribution based on the historical implied volatility series: low, moderate, and high. The daily dataset that I use starts on January 21, 1999 and ends on March 4, 2011. The mean of VIX is roughly 22 for the sample period. I define a moderate regime as plus/minus 0.5 standard deviation from its mean.

VIX Regimes	
Low :	$VIX_t < 17.02$
Moderate :	$17.02 < VIX_t < 26.95$
High :	$VIX_t > 26.95$

A simple non-stochastic GARCH(1,1) setting that allows changes in implied volatility is given by three mean equation equations:

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \sum_{i=1}^{3} \beta_i D_i logVix_t + \varepsilon_t;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \sum_{i=1}^{3} \beta_i D_i logVix_t + \alpha_3(\partial_k * 100 * \Gamma_{T-1})\mathrm{d}_{T-1}\mathrm{r}_{t-1} + \alpha_4(\partial_k * 100 * \Gamma_{T-1})(1 - \mathrm{d}_{T-1})\mathrm{r}_{t-1} + \varepsilon_t;$$

$$r_t = \mu_{t-1} + \sum_{i=1}^{2} \alpha_i r_{t-i} + \sum_{i=1}^{3} \beta_i D_i logVix_t + \alpha_3(100 * \Gamma_{T-1})\mathrm{d}_{T-1}\mathrm{r}_{t-1} + \alpha_4(100 * \Gamma_{T-1})(1 - \mathrm{d}_{T-1})\mathrm{r}_{t-1} + \varepsilon_t;$$

$$h_t = \omega + \theta\varepsilon_{t-1}^2 + \beta h_{t-1}.$$

where $i = L, M, H$ and $D_i = 1$ if $Vix_t\ belongs\ to\ the\ corresponding\ regime.\ D_i = 0,\ otherwise.$

III. Empirical Results

III.1. *Data and Descriptive Statistics*

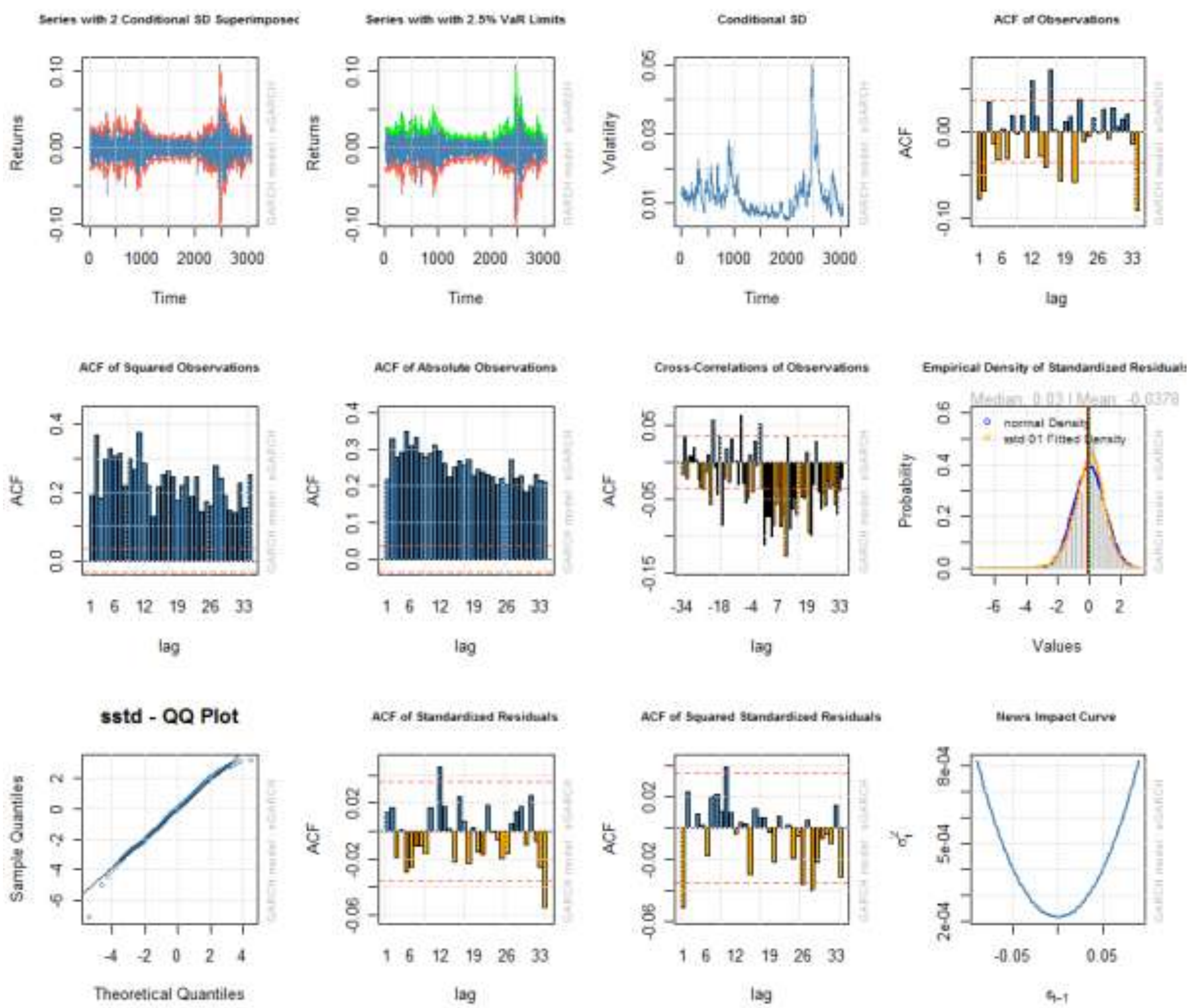
To capture the US stock market returns, I use the log-daily returns on S&P 500 index. The dataset covers three periods: For the benchmark model, it starts from December 29, 1998 to

March 4, 2011. For investor sentiment models, the monthly series starts from March 2, 2001 to December 31, 2010 and the weekly series starts from February 4, 1999 to March 4, 2011. Daily returns for S&P 500 are obtained from CRSP. Implied volatility (VIX) starts from December 29, 1998 to March 4, 2011 and is obtained from the Chicago Board Options Exchange (CBOE). Investor survey data is obtained from *American Association Individual Investors*, which is assumed to represent *individual* investor sentiment and from *Investors Intelligent*, which is assumed to represent *institutional* investor sentiment. Both surveys are taken weekly. *Table 15* provides the descriptive statistics for daily log returns and daily log VIX. The unconditional standard deviations of daily returns are in the range of 1.35% for the S&P 500 index and 6.07 % for the CBOE's VIX. Both the skewness and kurtosis measures indicate that the return and implied volatility distributions are not normal.

Table 15	*Descriptive Statistics*	
	Daily log-Returns	*Daily log-VIX*
Mean	0.00002	-0.00007
Median	0.00052	-0.00449
Std. Dev.	0.0135	0.0607
Maximum	0.1096	0.4960
Minimum	-0.0947	-0.3506
Skewness	-0.112	0.532
Kurtosis	7.359	3.906
Count	3066	3066

III.2. *Estimation Results*

The estimated coefficients of the GARCH models are reported in *Table 16*. I estimate a base model that excludes sentiment and implied volatility as explanatory variables in the mean equation (M0). The plots for the benchmark model are given below. The first three plots represent the series with two conditional distributions, 2.5% VaR limits and conditional standard deviation, respectively. They indicate that the returns have had high volatility during the 2000-dot-com crisis and the 2008-financial meltdown and have had low volatility during the expansion periods from 2003 to 2007. Autocorrelation function (ACF) suggests that returns are serially highly correlated during 30-day lags. The most dominant cross correlations occur somewhere between the lag one and 5 and then at lag 10. I use a skewed Student-t distribution for the estimation which fits the data better than normal distribution. The diagonal straight line of the sstd Q-Q plot, which shows quantiles of the data versus quantiles of the skewed Student-t distribution confirms that the skewed Student-t distribution fits the data well. The news impact curve measures the effect of a news at time t-1 on volatility at time t, while the information dated t-2 and earlier is held constant. The left (right) side of the chart shows how volatility responds to bad (good) news. It seems that the effects of good or bad news are not asymmetric.



Test results indicate that the mean reversion in daily returns is present and significant. A greater degree of mean reversion in high frequency data occurs when investors are pessimistic about the outlook of the economy, which creates high volatilities in prices. Confidence bounds around the point estimate for the mean reversion is between -0.05 and -0.12 percentage point (pp) for all models. Investors' bullish and bearish effects have the expected signs in mean reversion, consistent with the earlier hypothesis. For the effect of monthly investor sentiment on daily returns (MM0), average mean reversion is about -0.10 pp. Although bullish effect in the mean equation is insignificant, bullish (bearish) sentiment, on average, reduces (increases) the degree of mean reversion by 0.01 (-0.06) pp. This means that when investors are optimistic (pessimistic), daily returns mean-revert, on average, by -0.09 pp (-0.16 pp) throughout the next month. For the effect of weekly investor sentiment on daily returns (MW0), average mean reversion is about -0.06 pp. Bullish (bearish) sentiment, on average, reduces (increases) the degree of mean reversion by 0.01 (-0.04) pp throughout the next week, both of which are significant. Adding log changes in implied volatility into the mean equation increases goodness of fit in all specifications. A significant negative relationship between the index return and changes in implied volatility is found in all models (M1, MM1, MW1), which is roughly -0.12 pp. The results suggest that investors are, on average, penalized for the time-varying risk they take. When added in the mean equation, implied volatility insignificantly reduces (significantly increases) the effect of monthly (weekly) investor sentiment, which either suggests that those models explaining daily returns and volatility that use constantly updated (weekly) investor sentiment do a better job when compared to the models which use monthly series or that monthly selected macro economic variables, which are used in Chapter II, to explain the *observable* component of investor sentiment need a revision. Splitting the changes in log implied volatility, first defining them as *low*, *moderate*, and *high* volatility states, and then adding them as dummy variables in the mean equation improves the test results overall. The relationship between daily log returns and log changes in different-state implied volatility is negative and significant for all models (M2, MM2, MW2), which are, on average, roughly -0.09 pp, -0.14 pp, and -0.22 pp, respectively. As expected, the relationship is stronger for the high state compared to the low state. Under these models, the effect of bullish sentiment on mean reversion has almost gone to zero, whereas the effect of bearish sentiment has been reduced insignificantly (been increased significantly) for the monthly (weekly) series. For example, for the monthly series when investors are optimistic (pessimistic), daily returns mean-revert, on average, by -0.07 pp (-0.10 pp) throughout the next month. For the weekly series, when investors are optimistic (pessimistic), daily returns mean-revert, on average, by -0.10 pp (-0.18 pp) throughout the next week.

Table 16

	Benchmark (without Investor Sentiment)			Monthly Investor Sentiment (Indirect)			Weekly Investor Sentiment (Direct)		
	M0	M1	M2	MM0	MM1	MM2	MW0	MW1	MW2
μ_{t-1}	0.0004 (2.163)	0.0003 (2.756)	0.0005 (4.433)	0.0004 (1.710)	0.0003 (2.915)	0.0005 (4.154)	0.0004 (2.273)	0.0003 (2.796)	0.0005 (4.530)
r_{t-1}	-0.0714 (-4.703)	-0.0621 (-3.772)	-0.0571 (-3.229)	-0.1019 (-2.322)	-0.0875 (-3.878)	-0.0723 (-3.074)	-0.0983 (-6.263)	-0.1152 (-5.393)	-0.0989 (-4.369)
r_{t-2}	-0.0601 (-2.871)	0.0076 (1.038)	0.0143 (1.057)	-0.0618 (-2.342)	-0.0053 (-0.675)	0.0049 (0.254)	-0.0630 (-3.237)	-0.0067 (-0.509)	0.0019 (0.189)
$logVix_t$		-0.1255 (-23.138)			-0.1202 (-24.377)			-0.1254 (-23.196)	
$logVix_t^L$			-0.0943 (-28.083)			-0.0948 (-28.108)			-0.0949 (-27.731)
$logVix_t^M$			-0.1410 (-20.276)			-0.1371 (-17.925)			-0.1411 (-20.518)
$logVix_t^H$			-0.2222 (-14.152)			-0.2199 (-10.239)			-0.2223 (-13.438)
Bullish Effect				0.0183 (0.158)	0.0123 (0.621)	0.0067 (0.478)	0.0118 (2.483)	0.0199 (2.464)	0.0000 (1.860)
Bearish Effect				-0.0696 (-1.418)	-0.0555 (-1.503)	-0.0379 (-1.242)	-0.0304 (-2.236)	-0.0506 (-3.091)	0.0865 (-2.982)
ω	0.0000 (0.211)	0.0000 (0.191)	0.0000 (0.283)	0.0000 (0.184)	0.0000 (0.122)	0.0000 (0.221)	0.0000 (0.199)	0.0000 (0.194)	0.9107 (0.262)
ε_{t-1}^2	0.0747 (1.235)	0.0785 (1.141)	0.0823 (1.284)	0.0773 (1.086)	0.0879 (0.803)	0.0935 (0.971)	0.0747 (1.175)	0.0791 (1.145)	1.1545 (1.241)
h_{t-1}	0.9219 (15.886)	0.9182 (14.915)	0.9145 (16.750)	0.9185 (13.482)	0.9084 (9.523)	0.9022 (11.394)	0.9219 (15.114)	0.9175 (14.898)	5.6049 (15.526)
Skewness	0.8957 (42.155)	1.1083 (38.985)	1.1482 (35.607)	0.8899 (30.676)	1.1251 (37.234)	1.1659 (30.813)	0.8986 (40.644)	1.1124 (39.211)	35.894
Kurtosis	8.9083 (2.582)	7.5705 (3.924)	5.6450 (5.076)	9.0766 (2.074)	7.4433 (2.148)	5.2015 (3.844)	8.6738 (2.629)	7.4946 (3.534)	4.459
LLH	9550.46	10898.84	11029.41	7782.47	8884.68	8978.11	9481.25	10824.54	10947.83
# of obs	3,063	3,063	3,063	2,472	2,472	2,472	3,038	3,038	3,038

Note: *t-statistics are given in parenthesis*

The performance of GARCH specifications is examined by computing the distributional statistics of standardized (and squared) residuals for 20 lags along with an ARCH-LM test for 10 lags. If the model fits the data properly, we expect those residuals to be free of serial correlation and follow a normal distribution. However, as indicated earlier in Chapter II, I use a skewed Student-t distribution to capture the observed kurtosis in all the GARCH specifications. *Table 16* shows that the level of excess skewness and kurtosis have been significant in all specifications. The diagnostic test results in *Table 17* show that the specifications, when bullish and bearish effects are used, succeed in removing the serial correlations, whereas they fail to remove the serial correlations when changes in different-state implied volatility are included into the models.

Table 17	***Benchmark (without Investor Sentiment)***			***Monthly Investor Sentiment (Indirect)***			***Weekly Investor Sentiment (Direct)***		
	M0	M1	M3	MM0	MM1	MM3	MW0	MW1	MW3
Akaike	-6.231	-7.111	-7.195	-6.288	-7.179	-7.253	-6.235	-7.119	-7.199
$Q(20)$	22.60	22.63	*36.22*	17.28	14.85	21.27	23.11	20.19	*32.90*
$Q^2(20)$	23.98	7.25	*234.00*	26.83	10.96	*203.70*	24.22	7.25	*234.10*
ARCH Lag[10]	*18.66*	3.64	*242.30*	*24.16*	4.99	*211.70*	19.37	3.74	*243.60*
Adjusted Pearson	54.25	54.03	50.21	*81.60*	63.36	45.15	60.85	45.31	50.87

***Note**: Italicized estimations are significant at 5%*

IV. Chapter Conclusion

Studying survey-based measures of investor sentiment in GARCH models along with implied volatility (VIX) in high frequency series seem to provide results that are easy to explain. Using the Kahneman (1992) reference point idea, I first introduced a methodology which uses the estimated measures of sentiment to calculate investors' expectations, which are updated periodically and implemented in AR(1) process in the mean equation. Test results indicate that there is a positive (negative) insignificant (significant) relationship between daily index returns and bullish (bearish) sentiment. Using log changes in *implied* volatility (rather than conditional volatility as in a GARCH-in-mean setting) as an explanatory variable, I find a negative significant relationship between implied volatility and daily returns, which suggests that investors are, on average, penalized for the time-varying risk they take. The results also suggest that investors' pessimism (optimism) about the outlook of the economy, especially when implied volatility is included in the mean equation, increases (decreases) the effect of mean reversion in daily returns. This may suggest that periodic widespread pessimism causes prices to deviate from their fundamental values, which leads the market to get into temporary correction periods. The effects are more significant when we control the states (as *low*, *moderate*, and *high* volatility) of changes in log implied volatility, especially for weekly series where investors' expectations are calculated directly from the surveys.

V. Final Conclusion

Conventional asset pricing models mainly focus on formulating a relationship between stock market volatility and returns and ignore the effect of investors' expectations on prices, except those models which study noise trading where investor sentiment is viewed as irrational or simply a noise. However, in the last couple of decades, inspired by DSSW (1990), sentiment-based asset pricing models have gained popularity in the literature and have made significant contributions to the field. Whaley (2008), Giot (2005), and Copeland and Copeland (1999) were among others who studied the relationship between market volatility and returns and produced significant results explaining the role of *implied* volatility in returns. Moreover, conventional GARCH models introduced by Engle (1982) and Bollerslev (1986, 1987) have gained popularity explaining *volatility clustering* of financial data that exhibit excess kurtosis and tail thickness. Especially, GARCH-in-mean models which allow conditional volatility in the mean equation are designed to measure the effects of time-varying volatility on returns. In the attempt to measure the effect of investor sentiment on stock index returns, I use two surveys, namely changes in the *Bull Ratio* of AAII (for individual investors) and II (for institutional investors) along with a set of macroeconomic variables. I first determine what the *observable* component of sentiment is by regressing the surveys' *Bull Ratios* on selected macroeconomic variables and I call their residuals as *unobservable* component of the sentiment. Using GARCH-in-mean models, I found a positive and statistically significant relationship between changes in sentiment *Bull Ratio* of both investors and S&P 500 excess returns in the following month. The results indicate that the relationship is stronger for *institutional* investors suggesting that the opinions of *institutional* investors matter more relative to the opinions of *individual* investors. Instead of using conditional volatility, I repeat the same estimation for GARCH models. The estimated coefficients obtained from GARCH models which include exogenous *implied* volatility in the mean equation have similar signs and significance with GARCH-in-mean models but the effects of sentiment on excess returns were reduced, which is more obvious for the *observable* component of institutional investors. Repeating the both methodologies for weekly series, where I excluded selected macroeconomic variables and used surveys directly, I found consistent test statistics. In behavioral models, it is believed that investors' widespread optimism and pessimism can cause prices to deviate from their fundamental values, leading to temporary short or long term corrections in the form of mean-reverting behavior when those expectations are not met. Using periodic *realized* market returns as anchors, I define the difference between *expected* returns, which are estimated by monthly and weekly survey-based data, and realized returns as investor sentiment that is reflected on the daily AR(1) process, which measures the degree of mean reversion in each period. I call it bullish (bearish) sentiment if the difference is positive (negative). Test results indicate that there is a positive (negative) insignificant (significant) relationship between daily index returns and bullish (bearish) sentiment, which also suggests that investors' pessimism (optimism) about the outlook of the economy, especially when implied volatility is controlled (as low, moderate, and high state), can increase (decrease) the effect of mean reversion in daily returns.

References

1. **ALBERG**, D., **SHALIT**, H. and **YOSEF**, R 2006. "Estimating stock market volatility using asymmetric GARCH models". Discussion Paper No. 06-10.
2. **ALPERT**, Marc and Howard **RAIFFA** 1982. “A Progress Report on the Training of Probability Assessors,” in *Judgment Under Uncertainty,* Cambridge University Press. Pp. 294-305.
3. **BAKER**, M. and Jeffrey **WURGLER** 2004. “Market Liquidity as a Sentiment Indicator.” *Journal of Financial Markets,* 7(3): 271–99.

4. **BAKER**, M. and Jeffrey **WURGLER** 2006. "Investor sentiment and the cross-section of stock returns. *Journal of Finance*, 61(4), 1645–1680.

5. **BAKER**, M. and Jeffrey **WURGLER** 2007. "Investor Sentiment in the Stock Market". *Journal of Economic Perspectives*—Volume 21, Number 2—Spring 2007—Pages 129–151.

6. **BARBER**, Brad M. and Terrance **ODEAN** 2001. "Boys will Be Boys: Gender, Overconfidence, and Common Stock Investment," *Quarterly Journal of Economics*, Vol. 116(1), pp. 261-292.

7. **BARBERIS**, Nicholas, Andrei **SHLEIFER**, and Robert **VISHNY** 1998. "A Model of Investor Sentiment," *Journal of Financial Economics*, vol. 49, no. 3 (September):307-343.

8. **BARBERIS**, Nicholas and Richard **THALER** 2003. "A Survey of Behavioral Finance," NBER Working No. 9222.

9. **BLACK**, Fischer 1986. "Noise," *Journal of Finance*, 41(3), 529–543.

10. **BLACK**, Fischer; **SCHOLES**, Myron 1973. "The Pricing of Options and Corporate Liabilities". *Journal of Political Economy*81 (3): 637–654.

11. **BOLLERSLEV**, T., 1986. "Generalized autoregressive conditional heteroscedasticity". *Journal of Econometrics* 31(3), 307–327.

12. **BOLLERSLEV**, T., 1987. "A conditionally heteroscedastic time series model for speculative prices and rates of
13. return". *Review of Economics and Statistics* 69, 542–547.

14. **BOLLERSLEV**, T., 2008. "Glossary to ARCH (GARCH)". *Oxford University Press*, Oxford, UK.

15. **BROWN**, G.W., 1999. Volatility, sentiment, and noise traders. Financial Analysts Journal 55 (2), 82–90.

16. **BROWN**, G.W., and **CLIFF**, M. T. 2004. "Investor sentiment and the near-term stock market". *Journal of Empirical Finance*, 11(1), 1–27.

17. **BROWN**, G.W., and **CLIFF**, M. T. 2005. "Investor sentiment and asset valuation". *Journal of Business*, 78(2), 405–440.

18. **CHEN**, C.R., **SU**, Yuli, and **HUANG**, Y. 2007. "Hourly index return autocorrelation and conditional volatility in an EAR–GJR-GARCH model with generalized error distribution". *Journal of Empirical Finance*, 15, 789-798.

19. **CHOE**, Kwang-Il & **NAM**, Ki Seok and **VAHID**, Farshid, 2007. "Necessity of negative serial correlation for mean-reversion of stock prices". *The Quarterly Review of Economics and Finance*. 47, 576-583.

20. **COPELAND**, M. M. and **COPELAND**, T. E. 1999. "Market timing: Style and Size rotation using the VIX". *Financial Analysts Journal*. Mar/Apr 1999; 55, 2. Page 73.

21. **DANIEL**, Kent, David **HIRSHLEIFER**, and Avanidhar **SUBRAHMANYAM** 1998. "Investor Psychology and Security Market Under- and Overreactions." *Journal of Finance*, vol. 53, no. 6 (December): 1839-86.

22. **DE BONDT**, Werner F.M. and Richard **THALER** 1985. "Does the Stock Market Overreact?" *Journal of Finance*, vol. 40, no. 3 (July): 793-805.

23. **DE LONG**, Bradford, **SHLEIFER**, Andrei, **SUMMERS**, Lawrence, and **WALDMANN**, Robert J. 1990a. "Noise Trader Risking Financial Markets," *Journal of Political Economy*, vol. 98, no.4.

24. **DE LONG**, Bradford, **SHLEIFER**, Andrei, **SUMMERS**, Lawrence, and **WALDMANN**, Robert J. 1990b. "Positive Feedback Investment Strategies and Destabilizing Rational Speculation," *Journal of Finance* 45, 375-395.

25. **EDERINGTON**, Louis H. and Wei **GUAN** Fall 2002. "Measuring Implied Volatility: Is an Average Better? Which Average?" *Journal of Futures Markets*, Vol. 22 (9), 811-837.

26. **ENGLE**, R.F. 1982. "Autoregressive conditional heteroscedasticity with estimates of U.K. inflation". *Econometrica* 50. 987 1008.

27. **ENGLE**, R.F., **LILIEN**, D.M., and **ROBINS**, R.P., 1987. "Estimating time varying risk premia in the term structure: The ARCH-M model". *Econometrica* 55, 391–407.

28. **FAMA**, Eugene F. and Kenneth R. **FRENCH** 1988. "Permanent and Temporary Components of Stock Prices," *The Journal of Political Economy*, 96(2), 246–273.

29. **FISCHOFF**, Baruch; Paul **SLOVIC**, and Sarah **LICHTENSTEIN** 1977. "Knowing with certainty: The appropriateness of extreme confidence". Journal of Experimental Psychology: Human Perception and Performance **3**: 552–564.

30. **FISHER**, K. L., and **STATMAN**, M. 2000. "Investor sentiment and stock returns". *Financial Analysts Journal*, 16–23.

31. **FRENCH**, K.R., **SCHWERT**, G.W., and **STAMBAUGH**, R. F. 1987. "Expected stock returns and volatility". *Journal of Financial Economics* 19, 3–29.
32. **FULLER**, Russell J. 1998. "Behavioral Finance and the Sources of Alpha," *Journal of Pension Plan Investing*, vol. 2, no. 3.
33. **GERVAIS**, S and **ODEAN**, T. 2001. "Learning to be Overconfident". *Review of Financial Studies*, Spring 2001, Vol. 14, No. 1, pp. 1-27.
34. **GIOT**, P. 2005. "Implied Volatility Indexes and Daily Value at Risk Models". *Journal of Derivatives*. 12, 54-64.
35. **GLOSTEN**, L.R., **JAGANNATHAN**, R., and **RUNKLE**, D.A., 1993. "On the relation between the expected value and the volatility of the nominal excess return on stocks". *Journal of Finance* 48 (5), 1779–1801.
36. **GRAY**, S.F., 1995. "An analysis of conditional regime-switching models", Working paper (Fuqua School of Business, Duke University, Durham, NC).
37. **GRAY**, S.F. 1996. "Modeling the conditional distribution of interest rates as a regime-switching process". *Journal of Financial Economics*. 42, 27-62.
38. **GRIFFIN**, D.W. and Amos **TVERSKY** 1992. "The Weighing of Evidence and the Determinants of Confidence," *Cognitive Psychology,* 24, 411-435.
39. **GROSSMAN**, Sanford J., and **SHILLER**, Robert J. 1981. "The Determinants of the Variability of Stock Market Prices."*The American Economic Review*, Vol. 71, No. 2, (May 1981), pp. 222-227.
40. **HAMILTON**, J.D., 1989. "A new approach to the economic analysis of nonstationary time series and the business cycle". *Econometrica* 57, 357 384.
41. **HUBERMAN**, Gur and Tomer **REGEV** 2001. "Contagious Speculation and a Cure for Cancer: A Non-Event that Made Stock Prices Soar In," *Journal of Finance,56(1), pp. 387-96.*
42. **KAHNEMAN**, Daniel, and Amos **TVERSKY** 1979. "Prospect Theory: An Analysis of Decision under Risk." *Econometrica*, vol. 46, no. 2 (March):171-185.
43. **KAHNEMAN**, Daniel, and Amos **TVERSKY** 1981. "The framing of decisions and the psychology of choice, *Science 211*, 453-458
44. **KAHNEMAN**, Daniel, and Amos **TVERSKY** 1982. *Judgment under uncertainty: Heuristics and biases*. New York: Cambridge University Press.
45. **KAHNEMAN**, Daniel, and Amos **TVERSKY** 1992. "Advances in prospect theory: Cumulative representation of uncertainty" *Journal of Risk and Uncertainty*, 5(4), 297–323.
46. **KAROLYI**, G.A. 2001. "Why Stock Return Volatility Really Matters. Prepared for Inaugural Issue of Strategic Investor Relations". (published by *Institutional Investor Journals Series*).
47. **LAMBERT**, P., **LAURENT**, S. and **VEREDAS**, D. 2007. "Testing conditional asymmetry: A residual-based approach". Working Paper.
48. **LAMBERT**, P., and S. **LAURENT** 2001a. "Modeling Financial Time Series Using GARCH Type Models and a Skewed Student Density," Discussion Paper 01-25, Institut de Statistique, Louvain-la-Neuve.
49. **LAMBERT**, P., and S. **LAURENT** 2001b. "Modeling Skewness Dynamics in Series of Financial Data Using Skewed Location-scale Distributions," Discussion Paper 01-19, Institut de Statistique, Louvain-laNeuve.
50. **LAMOUREUX**, C.G. and W.D. **LASTRAPES**, 1990. "Persistence in variance, structural change, and the GARCH model". *Journal of Business and Economic Statistics* 8, 225 234.
51. **LEE**, W. Y., **JIANG**, C. X., and **INDRO**, D. C. 2002. "Stock market volatility, excess returns, and the role of investor sentiment". *Journal of Banking & Finance*, 26, 2277–2299.
52. **LEROY**, Stephen F. 1989. "Efficient Capital Markets and Martingales," *Journal of Economic Literature*, 27(4), 1583–1621.
53. **LUCAS**, Robert E., Jr., 1978. "Asset Prices in an Exchange Economy," *Econometrica*, 46(6), 1429–1445.
54. **MALKIEL**, Burton G. 2003. "The Efficient Market Hypothesis and Its Critics," *Journal of Economic Perspectives*, 17(1), 59–82.
55. **MANDELBROT**, Benoit 1963. "The Variation of Certain Speculative Prices," *The Journal of Business*, 36(4), 394–419.
56. **MARKOWITZ**, Harry M. March 1952. "Portfolio Selection," *The Journal of Finance* 7 (1): 77–91.
57. **MARKOWITZ**, Harry M. 1959. *Portfolio Selection: Efficient Diversification of Investments*. New York: John Wiley & Sons. (reprinted by Yale University Press, 1970, 2nd ed. Basil Blackwell, 1991

58. **ODEAN**, Terrance 1998. "Are Investors Reluctant to Realize Their Losses?" *Journal of Finance*, vol. 53, no. 5 (October):1775-98. ---1998b. "Volume, Volatility, Price and Profit When All Traders Are Above Average." *Journal of Finance*, vol. 53, no. 6 (December): 1887-1934.
59. **POTERBA**, James M. and Lawrence H. **SUMMERS** 1988. "Mean Reversion in Stock Prices: Evidence and Implications," *Journal of Financial Economics*, 22, 27–59.
60. **SHEFRIN**, Hersh and Meir **STATMAN** 1985. "The Disposition to Sell Winners Too Early and Ride Losers Too Long: Theory and Evidence*," Journal of Finance*, vol XL, No 3.
61. **SHLEIFER**, Andrei and **SUMMERS**, Lawrence H. 1990. "The Noise Trader Approach to Finance," *The Journal of Economic Perspectives*.
62. **SHLEIFER**, Andrei and Robert **VISHNY** 1997. "Limits of Arbitrage." *Journal of Finance*, vol. 52, no. 1 (March): 35-55.
63. **SUMMERS**, Lawrence H. 1986. "Does the Stock Market Rationally Reflect Fundamental Values?" *The Journal of Finance*, 41(3), 591–601.
64. **VERMA**, R. and **SOYDEMIR**, G. 2009. "The impact of individual and institutional investor sentiment on the market price of risk". *The Quarterly Review of Economics*, 49, 1129-1145.
65. **WHALEY**, Robert E., 1993, Derivatives on market volatility: Hedging tools long overdue, Journal of Derivatives 1, 71-84.
66. **WHALEY**, Robert E. 2008. "Understanding VIX". Working Paper.
67. **WURTZ**, D. Y. **CHALABI** and L. **LUKSA** 2007. "Parameter Estimation of ARMA Models with GARCH/APARCH Errors An R and SPlus Software Implementation". *Journal of Statistical Software* (forthcoming).