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Transaction costs and nonlinear adjustment in the Brazilian Real's real exchange rates in the post-Real plan era. Revisiting the PPP puzzle

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Abstract

The present work intends to model the dynamics of the adjustment process to long run purchasing power parity (PPP) of the Brazilian real relative to a number of currencies, by employing a univariate non-linear model that incorporates structural breaks following the general equilibrium model developed by Dumas (1992). The objective of this dissertation is to revisit past empirical works on the PPP doctrine by applying a non-linear ESTAR to model the adjustment process of the Brazilian real's real exchange rate series, to test for its mean reverting properties, and account for the strong persistence observed in the post floating era. By incorporating transaction costs and other barriers to international trade, it is possible to reconcile the general Casselian view of PPP with the mixed evidence presented in past empirical works. This study finds evidence of non-linear mean reverting behaviour for the real's real exchange rate series, which explains the failure of conventional linear cointegration tests in supporting long-run PPP.

Keywords: Non-linearities; Real Exchange Rates, STAR models, Purchasing Power Parity.

1. Introduction

The long-run Purchasing Power Parity (PPP) relationship remains the major theoretical building block of most open macroeconomic models. In its absolute form, the PPP condition asserts that the same bundle of goods, measured in real terms, should have the same value across countries.[1] However, this theoretical relationship between nominal exchange rates and cross country price differentials rests on the assumptions of the absence of factors that may hamper or impede international goods arbitrage. Whether it be the presence of transaction costs, the existence of nontradable goods, market structures characterized by imperfect competition or the implementation

[1] It is worthwhile to note that it is theoretically possible for the real exchange rate to be in equilibrium at a level other than the PPP level. In open economic general equilibrium models, the real exchange rate is determined endogenously as the result of optimizing behaviour by agents in clearing markets with rational expectations (see Stockman, 1980; Lucas, 1982).

of any other barrier to trade with similar effects to tariffs or quotas, the influence of these factors on the equilibrium parity condition are analogous. The existence of such market frictions, that in effect limit arbitrage opportunities, would imply that the PPP hypothesis may only be plausible as a long run relationship.

For all of the above reasons, it is highly unlikely that the PPP hypothesis holds as an equilibrium parity condition in its traditional representation. The prevailing evidence based on a linear cointegration framework is at best inconclusive (see for example, Frankel and Rose, 1996; Lothian and Taylor, 1996; Sarno and Taylor, 2002; Cerrato and Sarantis, 2003), and the empirical failure to find support of the PPP hypothesis has been termed the PPP puzzles (Rogoff, 1996; Obstfeld and Rogoff, 2001). The first paradox arises from the non-stationary properties of real exchange rates, which do not converge to PPP equilibrium and exhibit mean-averting behaviour. Even when studies have employed a larger sample size, and have successfully found evidence of long-run mean reverting behaviour for PPP deviations, the considerable persistence displayed by the series cannot be accounted for by the present theoretical building block. The high persistence manifested in real exchange rate series, measured in terms of half-lifes, constitutes the second PPP puzzle. Ultimately, the shortcomings of the PPP hypothesis still remain to date one of the most controversial fields in international macroeconomics.[2]

Over the past decades, several researchers have argued that the inability of finding mean reverting behaviour in real exchange rate series could be a result of the presence of nonlinearities in the data generating process, which are not considered in traditional testing procedures. Consequently, conventional tests have necessarily lower power in rejecting mean-averting behaviour, which implies that the conclusions drawn from past studies may prove to be invalid. One of the most notable theoretical explanations to why convergences to long-run PPP may be nonlinear is based on

[2]Following an alternative strand of work, stronger evidence of mean reversion in real exchange rate series has been found by employing panel cointegration tests (see for example, Wu, 1996; Papell and Theodoridis, 1998). However the use of such unit-root test variants have been deemed inappropriate since they fail to control for cross-sectional dependence in the data (O'Connell, 1998).

the presence of transaction costs in spatially separated markets. Such structural models could account for the persistence of real exchanged rate series observed in the post Bretton-Woods era, while still validating the PPP hypothesis as an equilibrium relationship for real exchange rates. The works by Dumas (1992), Uppal (1993) and Sercu et al.(1995) develop general equilibrium models for small open economies, in which the existence of transport costs in commodity goods arbitrage generate a nonlinear adjustment process of real exchange rates towards PPP equilibrium. The natural presence of market frictions in commodity trade across countries (for example iceberg costs) generates a neutral band within which arbitrage is not profitable. This implies that for small deviations from PPP equilibrium forces are not at play, and the real exchange rate necessarily follows an atypical nonlinear process.

In order to test the hypothesis of nonlinear convergence to PPP equilibrium based on market frictions, the use of the Smooth Transition Autoregressive (STAR) was initially adopted in the works of Michael et al. (1997). Following this line of research, several authors have found strong evidence in support of nonlinear mean reversion for PPP deviations (see for example, Chen and Wu, 2000; Baum, Barkoulas and Caglayan, 2001; Kapetanios, Shin and Snell, 2003; Sollis, 2008). The evidence obtained from these studies have shed light into the dynamics of real exchange rate series, and more importantly posits that there still remains hope for PPP as a theoretical building block in exchange rate determination.[3]

In the present study, the objective is twofold: firstly to revisit past empirical works on relative PPP by applying a non-linear STAR to model the adjustment process of the Brazilian *real*'s real exchange rate series in order to analyse its mean reverting properties. Secondly, to elucidate the persistence of the *real's* real exchange rate series given its nonlinear convergence to PPP equilibrium. The choice of focusing on the *real* comes from the growing presence of the Brazilian

[3] There exists already significant evidence of nonlinearities in deviations from PPP (Michael et al. 1997; Sarantis, 1999; Baum et al., 2001; and Taylor et al., 2001), which would warrant the use of newly developed testing procedures to find evidence of cointegration. The main drawback from this strand of work is that the series are assumed to be stationary under the null, when carrying out test hypothesis for linearity in the series.

economy in global markets since the stabilization program introduced in 1994. The results obtained from the real exchange rate series provide an indicator of competitiveness for the Brazilian economy, and carry significant policy implications. For instance, a non-stationary real exchange rate series ultimately invalidates the PPP hypothesis, and as such it cannot be used to determine equilibrium exchange rates.[4] From there on, there would be a valid case for exchange rate intervention with the purpose of stabilizing foreign exchange markets or harnessing competitive forces in foreign goods markets. Finally, the case of Brazil allows for a natural experiment for the purposes of the work, as the Brazilian economy stabilized following the Real Plan of 1994 and their currency has been following an appreciating trend for the past decade.

The rest of the paper is structured as follows. In section 2 a detailed explanation is given to why nonlinearities may arise in the dynamics of real exchange rates, and the general form of the nonlinear STAR model is presented. This section includes the testing procedure, which involves the following: a sequence of tests to assert the validity of the PPP hypothesis, testing the linearity hypothesis of the real exchange rate series, and distinguish the valid form of the STAR model. The estimation and results are presented in section 3. Finally, a summary and some concluding remarks are presented in the last section.

2. Nonlinear Adjustment of real exchange rate series towards PPP

In the context of a dynamic general equilibrium model with spatially separated markets, Dumas (1992), Uppal (1993) and Sercu et al., (1995) reveal that the existence of proportional transaction costs in international trade inexorably results in deviations from PPP converging to their equilibrium values in a nonlinear manner. The presence of transaction costs or any other barriers to trade generate a band of inaction in which deviations from PPP are not corrected due to the fact that

[4]Despite the candidacy of a myriad of international parity conditions that can be used for testing equilibrium exchange rate determination, the choice of the PPP relationship is preferred for the present case, as it measures the extent of integration between goods and foreign exchange markets across countries. Brazil adopted large protectionist policies during the 80s and beginning of 90s, which limited the trade openness of the domestic markets, hence the transaction costs are expected to be large in relation to the developed economies considered.

goods arbitrage is not profitable. Provided that the deviations are sufficiently large (so that the marginal profit of conducting international goods arbitrage exceeds the marginal cost), then the real exchange rate will necessarily exhibit nonlinear mean reverting behaviour. Subsequently, this family of models may prove to be a viable candidate in accounting for the failure of prior empirical evidence to support the PPP hypothesis, and can conceivably shed light into the PPP Puzzles (Rogoff, 1996).[5]

In Dumas' (1992) model, which will be the basis of this work, the case considered is that of two small open economies producing one homogenous good that can be invested, consumed or traded in international markets. The economies are subject to random productivity shocks and transferring goods from one country to another to smooth consumption is costly in terms of transaction or shipping costs. To illustrate this point the model assumes that out of one unit of good which is internationally traded, only(1-s), s$\in$(0,1) of the good actually arrives to the foreign market. Hence, it is only optimal for agents to rebalance their consumption plan after a productivity shock if the marginal benefit of incurring in international transactions exceeds the marginal costs of doing so, given the fact that there exists a shipping cost. As mentioned above, this creates a band around the PPP equilibrium in which no arbitrage occurs, and within this rage the exchange rate can deviate from its parity value and exhibit random walk like behaviour. Solely when deviations from parity are sufficiently large, does the exchange rate adjust to equilibrium value. This further implies that deviations from PPP will necessarily last a long time, however this does certainly not signify that they follow a random walk.

To model the dynamic behaviour of the real exchange rate, a variant of the *Smooth Transition Threshold Autoregressive (STAR)* model as proposed by Granger and Teräsvirta (1993), Teräsvirta (1994), Michael et al. (1994), is adopted. In this framework, there is *central regime* (in which no adjustment takes place, i.e. the value is located in its PPP equilibrium) and an *outer regime* (in

[5]See Sarno and Taylor (2002) for a survey of empirical literature related to this topic.

which mean reverting forces appear), and transition between regimes is smooth as opposed to discrete. As argued by Terasvirta, the existence of heterogeneous economic agents making dichotomous decisions, when not acting simultaneously, provides justification for modelling regime changes in a continuous manner rather than discrete. This framework has been employed by several authors (Michael, Nobay and Peel, 1997; Chen and Wu, 2000; Baum, Barkoulas and Caglayan, 2001), to find supporting evidence of the PPP hypothesis.[6] In addition, if the real exchange rate is constructed using price indices consisting of goods each with a distinct transaction costs, one would expect the adjustment process of the real exchange rate to be smooth and not discontinuous (Taylor, Peel and Sarno; 2001); hence the choice of the STAR model.

The requirement of symmetry in response to deviation from the PPP, suggests that a more specific variant of the STAR model can be employed by introducing an exponential transition function, which gives rise to the ESTAR model variant.[7] The rational for choosing a symmetric transition function can be explain by the fact that one would expect equilibrium forced to work identically irrespectively of the sign of the deviation. That is, arbitrage is considered equally profitable when the exchange rate is either appreciated or depreciated in real terms. Hence, this model can be considered as a generalization of the Threshold Autoregressive (TAR) model (Tong 1990) and its use seems warranted when modelling deviations from PPP. The advantage of employing this estimation procedure as opposed to the simpler linear cointegration framework is that it allows for non-constant speed of convergence to parity.[8] This signifies that the speed of adjustment is directly

[6]O'Connell (1998) applies a TAR model and finds that by only taking market frictions into account, the observed persistence in the series cannot be explained. His work strongly disputes the evidence presented by several authors (Lothian, 1997, Wu, 1996; and Papell and Theodoridis, 1998), in favour of the PPP hypothesis.

[7]Theoretical models in which this paper is based rely on the presence of proportional transport costs, which predict a symmetric adjustment process for real exchange rate series since the incentives for arbitrage work both ways. Hence the use of an ESTAR model seems more plausible. Nevertheless, in models where transport costs are non-proportional and economies of scale are present with different consumer preferences, asymmetric behaviour is likely to arise (Goswami et al., 2002). Such kinds of models are best characterized by an LSTAR model with a logistic transition function.

[8]The conventional cointegration framework assumes that deviations from PPP follow a linear process, which implies that the adjustment process, whilst being continuous, has constant speedy of adjustment which does not seem reasonable. Hence the existence of nonlinear adjustment to parity in the data generating process has strong implications on standard cointegration tests (van Dijk and Teräsvirta, 2001).

related with the size of the deviation from parity conditions, endowing the model with greater flexibility and explanatory capacity.

The maintained supposition that the STAR specification can prove to be more suitable for analyzing the dynamic behaviour of PPP deviation is supported by two further reasons. Firstly, as Teräsvirta (1994) points out, in Dumas'(1992) model transitions between regimes are smooth rather than discrete, which seems to be more appropriate for handling aggregate data in the presence of heterogeneous agents. In addition, the TAR model has been known to cause problems when testing for linearity, due to the discontinuity in the adjustment process at each of the thresholds, and fails to provide any clear guidance on how inference should be performed on the estimated thresholds (Tsay, 1989). Secondly, in relation to the previous argument, the analytical and statistical tools have been more cultivated for this family of models, making the analysis richer.

2.1 STAR family of models

The model considered in the previous subsection provides a natural and coherent method to model deviations from PPP $\{y_t\}$ in the presence of spatially separated markets with transaction costs. Aside from being able to incorporate market frictions to the adjustment process, the ESTAR model also posits that the adjustment towards PPP is independent of the sign of the deviation from equilibrium. The general specification of the ESTAR model for PPP deviations, that is the real exchange rate, can be represented in the following way

$$y_t = k + \sum_{j=1}^{p} \pi_j\, y_{t-j} + \left(k^* + \sum_{j=1}^{p} \pi^*\, y_{t-j}\right) \times F(\gamma, y_{t-d}) + u_t \qquad (1)$$

where y_t is a stationary and ergodic process, the error term u_t~n.i.d$(0,\sigma^2)$ and $\gamma>0$. The ESTAR model relies on the specification of the transition function $F(\gamma, y_{t-d})$, which implicitly determines the degree of mean reversion, and is itself dependent on the parameter γ which governs speed of the transition in the process. Since deviations from PPP are expected to be corrected symmetrically, the exponential specification appears appropriate due to its U-shape around the threshold value. The transition between the two extreme regimes is smooth in the sense that as the delay parameter increases in absolute value, the behaviour of the real exchange rate becomes more dependent upon the starred coefficients' values. Generally, as in the case of this paper, the transition variable itself is assumed to be a lagged endogenous variable (Teräsvita, 1994), which must be determined before proceeding with testing and estimating the model. The transition function in its exponential representation can be formulated as follows:

$$F(\gamma, y_{t-d}) = 1 - \exp\left[-\gamma(y_{t-d} - c^{*})^2\right] \tag{2}$$

and exhibits the following properties: F:(0,∞) → [0,1]; that is, the transition function is uniformly bounded between zero and one. In addition, it holds that F(0) = 0 and $\mathrm{Lim}_{x\to\pm\infty}F(x) = 1$. Moreover it is symmetric around the threshold value c*, hence the U-shape.

The arguments of the exponential function can be scaled by the sample variance of the transition variable following the suggestion of Granger and Teräsvirta (1993, p124). The scaling speeds the convergence and improves the stability of the nonlinear least squares estimation algorithm making estimation easier, and more importantly it allows the comparison of the estimates of γ across equations. Moreover, the scaling further simplifies the estimation process, by allowing γ=1 to be an appropriate starting value. The scaled exponential form of the transition function takes the following specification:

$$F(\gamma, y_{t-d}) = 1 - \exp\left[\frac{-\gamma(y_{t-d} - c^{*})^2}{\sigma_{yt}^2}\right] \tag{3}$$

Inserting equations (2) or (3) as the transition function in equation (1) leads to the ESTAR model representation. The depicted model presents the following desirable characteristic: when $y_{t-d}=c^{*}$, its equilibrium value, then the transition function yields F(.)=0 and the model reverts to a linear AR(p) representation. This corresponds to the inner regime displayed below:

$$y_t = k + \sum_{j=1}^{p} \pi_j\, y_{t-j} + u_t \tag{4}$$

For extreme deviations from PPP equilibrium value, that is when $y_{t-d \to \pm\infty}$, the transition function tends to F(.)=1 and the model reverts to a distinct AR(p) representation, with intercept k+k* and jth term $\pi_j + \pi_j^*$ as parameter coefficients. This refers to the outer regime with the following representation:

$$y_t = (k + k^*) + \sum_{j=1}^{p} (\pi_j + \pi_j^*) y_{t-j} + u_t \tag{5}$$

Therefore, it is straight forward to see that the model may well allow for $\sum_{j=1}^{p} \pi_j \geq 1$, implying that it is possible for the series to exhibit random-walk like or explosive dynamic behaviour for small deviations from PPP equilibrium. However, as long as the condition $\sum_{j=1}^{p} (\pi_j + \pi_j^*) < 1$, is satisfied, the model is globally stable and displays mean reverting properties. More importantly, as long as $\pi^* \neq 0$ for any j, then the outer regime has different speed of mean reversion.

In the subsequent analysis, a convenient reparameterisation of the model is carried out to convert the model into the ensuing error correcting form (Michael et al. 1997).

$$\Delta y_t = k + \lambda y_{t-1} + \sum_{j=1}^{p-1} \varphi_j \Delta y_{t-j} + \left(k^* + \lambda^* y_{t-1} + \sum_{j=1}^{p-1} \varphi_j^* \Delta y_{t-j}\right) \times F(\gamma, y_{t-d}) + u_t \tag{6}$$

The adjustment process of the real exchange rate towards PPP will depend on the coefficients of the model, where for small deviations from PPP, the transition function approaches zero, and the coefficient λ will govern the dynamic process. On the other hand, for large deviations from PPP equilibrium, F(.) approaches unity, and the coefficient λ* comes into play in the process. Interestingly, the representation of the model is equivalent to a standard linear Dickey-Fuller regression in the absence of nonlinearities (i.e. when γ=0) which is represented by

$$\Delta y_t = k + \lambda' y_{t-1} + \sum_{j=1}^{p-1} \varphi_j \Delta y_{t-j} \tag{7}$$

The most notable feature of the model is that if the real exchange rate series are close to equilibrium conditions, then the series may well display random-walk like behaviour, which is particularly true for those currencies that have stabilized in the managed- floating period. Moreover, the fact that the data generating process conceivably follows an ESTAR model has a direct implication for the validity of conventional cointegration tests based on linearity assumptions and Dickey-Fuller type regressions. Considering that model (6) is indeed the correct specification, then by adopting the ADF model in equation (7) the estimates of the parameter λ' will be some linear combination of λ and λ^*. This implies that the model's estimate of the impact of the lagged values of y_t will be inconsistent due to a problem of misspecification. Consequently, if the true process is given by a nonlinear model of the type considered, then the null hypothesis of no linear cointegration may not be rejected against the stationarity alternative, even if the true process is globally stable (Michael et al., 1997). Hence failure to reject the null of no linear cointegration by traditional testing procedures cannot be regarded as evidence against the PPP doctrine (Michael et al., 1997; Taylor et al., 2001).

2.2. Test Procedure

Following the procedure as proposed by Taylor, Peel and Sarno (2000) and Baum, Barkoulas and Caglayan (2001), prior to estimating the model, a series of tests should be carried out on the real exchange rate series to determine whether they are suitably represented by a univariate two-regime STAR model. Firstly, by employing both *Engle and Granger's two step procedure* and *Johansen's Methodology,* a linear cointegration test should be conducted as a way of testing for long-run PPP. Secondly, conditional on having found no conclusive evidence supporting the PPP hypothesis, the estimated disequilibrium series are subsequently subject to a test for linearity versus an STAR specification based on the artificial regression presented by Teräsvirta (1994).

The reason for carrying out the tests is that estimation of STAR model requires prior rejection of the null hypothesis of nonlinearities in the series considered. Furthermore, evidence of nonlinearities being present in the data generating process could account for the low power in standard unit root tests when discriminating against long-run PPP (Pippenger and Goering, 1993; Lothian and Taylor, 1996).

2.2.1 Test for cointegration.

The PPP hypothesis can be tested using the following regression:

$$e_t = c + \beta_1 P_t + \beta_2 P_t^* + y_t \qquad (8)$$

where C is a constant reflecting errors in unit of measurement, e_t is the log of the nominal exchange rate at time t, P_t and P_t^* are the logs of the domestic and foreign price levels respectively, at time t. And y_t is the disturbance that captures deviations from PPP, i.e. the real exchange rate. The PPP hypothesis in absolute form holds if, given the restriction β_1=-β_2=1(symmetry and proportionality), the errors in the series are stationary. However, in the presence of transaction costs and measurement errors in price indices, the sufficient condition for relative PPP demands that $\beta_1>0$ and $-\beta_2>0$ are satisfied, in addition to the error terms being stationary (Cheung and Lai, 1993a).[9]

Asserting the validity of the PPP doctrine as a stable long run relationship between nominal exchange rates and price differentials of domestic to foreign price levels for a given bundle of goods, requires that the errors from the cointegrating equation be stationary. Recent studies demonstrate that by employing Johansen's cointegration framework, evidence supporting PPP can be found from a smaller sample size without having to resort to panel data analysis (Baum et al., 2001). Any favourable evidence of mean reversion in the series should be taken as a first step to constructing an STAR model to capture the dynamics of PPP deviations, however as explained in the subsequent subsection, the results should be interpreted with caution. [10]

[9]The absolute version of PPP is not expected to hold in empirical work due to measurement errors which arise due to differences across countries in the way price indices are composed.

[10]The conclusions drawn from the cointegration analysis should be taken prudently for one further reason, as the Johansen test is known to be subject to finite sample bias towards over-rejection of the null hypothesis of no cointegration (Cheung and Lai;1993b).

2.2.2. Test for nonlinearity and the form of linearity.

Conditional on the series of deviations from PPP obtained after having imposed the parameter restrictions of symmetry and proportionality, a test for nonlinearity applies the artificial regression proposed by Teräsvirta (1994) in order to discriminate between a linear specification in favour of a STAR alternative. The following sequence of tests is known as *Teräsvirta's model selection criterion* which relies on the nature of the transition function.[11]

Testing for linearity

The transition function in its exponential form (2) implies that F(.)=0 when $\gamma=0$, hence the linearity hypothesis can naturally be tested as $H_0:\gamma=0$ against $H_1:\gamma>0$. However, equation (1) is not identified under the null hypothesis H_0 since, in the case $\gamma=0$ is true, then the parameters c^*, k^* and π_j^* for all j=1,2... can take any value as long as their averages remain the same. To address this problem, Luukkonen et al. (1988) point out that if the delay parameter (d) is kept fixed, a low-order Taylor approximation of the transition function around zero can be taken ($\gamma=0$) in order to obtain the unrestricted form of the model. The following artificial regression developed by Teräsvirta (1994) can be estimated by ordinary least squares (OLS):

$$y_t = \beta_{00} + \sum_{j=1}^{p}(\beta_{0j}y_{t-j} + \beta_{1j}y_{t-j}y_{t-d} + \beta_{2j}y_{t-j}y_{t-d}^2 + \beta_{3j}y_{t-j}y_{t-d}^3) + \eta_t \qquad (9)$$

From this point, a test for linearity can be carried out, and given the rejection of the null hypothesis of linearity, the above artificial regression can also be used to discriminate between a logistic and an exponential specification of the model (Saikkonen and Luukkonen, 1988; Luukkonen et al., 1988; Granger and Teräsvirta, 1993). The approach draws two main advantages. Firstly, the model does not need to be estimated under the alternative hypothesis, and secondly asymptotic critical values

[11] Taylor and Peel (2000) find that in some cases the use of a logistic specification is warranted. In fact Cerrato and Sarantis(2006) conclude that in roughly half of the emerging countries analyzed in their work, there is strong evidence of asymmetric behaviour in the mean reversion process for real exchange rates.

are available from standard asymptotic theory which makes the testing process manageable (van Dijk, Teräsvirta and Frances, 2000).

Testing for nonlinearity is equivalent to performing the following test hypothesis for equation (9) H_{0L}:β_{1j}=β_{2j}=β_{3j}=0 for j=1,2,...,p, against the alternative H_{1L}. It can be shown that the test statistic for the exclusion restrictions on the nonlinear terms are asymptotically equivalent to Lagrange Multiplier test statistics for STAR processes with χ^2 distributions (Luukkonen et al., 1988; Eitrheim and Teräsvirta, 1996). Nevertheless, in practice F based tests should be employed in order to improve size and power properties when faced with a small sample (Granger and Teräsvirta, 1993). This is particularly important when the number of observations is limited, yet the order of the linear autoregression(p) is relatively large.

Determining the transition variable (delay parameter)

Fortunately by applying the Taylor approximation, the appropriate transition variable in the STAR model can be determined without specifying the specific form of the transition function. The value of the delay parameter for y_{t-d} is chosen by computing the F statistic in the linearity test, for several candidate transition variable (say d=1,2,3,...dmax), and selecting the integer d for which the p-value of the test statistic is the smallest (Tsay, 1989). Since monthly data is being employed, it seems reasonable to set d.max=12. The logic behind this procedure is that the test should have the maximum power to discriminate, which implies that the alternative model is correctly specified (van Dijk, 1999).

Selecting the transition function

Ascertaining the form of the transition function $F(y_{t-d}; \gamma,c)$ is the final step prior to estimating the model. Following Teräsvirta's model selection criteria, which is essentially a decision rule based on a sequence of tests nested within the null hypothesis corresponding to the F statistic for the artificial regression (9), one is able to determine the correct specification of the transition function.

The proposed sequence of test hypotheses is as follows:

H_{01}: $\beta_{3j}=0$,

H_{02}: $\beta_{2j}=0|\beta_{3j}=0$,

H_{03}: $\beta_{1j}=0|\beta_{2j}=\beta_{3j}=0$, for all j=1,2,…,p

where both the LM type test and the finite sample F test are employed. If H_{01} is rejected, then LSTAR model should be selected. On the other hand, accepting H_{01} and rejecting H_{02} imply that the ESTAR model is the appropriate choice. However, by accepting both H_{01} and H_{02}, but rejecting H_{03}: then once again an LSTAR model is deemed suitable. However Teräsvirta (1994) and van Dijk and Teräsvirta (2000) argue that the strict application of this sequence of tests can draw wrong conclusions. Therefore, one should select the correct specification of the transition function on the basis of the lowest p-value associated to each individual test hypothesis, that is if the rejection of H_{02} displays the lowest p-value, then an ESTAR model should be chosen.

In addition, Teräsvirta's criteria can be complemented with an alternative decision rule for selecting the transition function proposed by Escribano and Jorda (1999). The authors claim that a lower-order Taylor approximation is not sufficient to capture the characteristics of the model (more specifically the inflection points of the function). Hence, the suggestion is to take a higher order Taylor expansion of the following form:

$$y_t = \beta_{\mathbf{00}} + \sum_{j=1}^{p}(\beta_{\mathbf{0}j}y_{t-j} + \beta_{\mathbf{1}j}y_{t-j}y_{t-d} + \beta_{\mathbf{2}j}y_{t-j}y_{t-d}^2 + \beta_{\mathbf{3}j}y_{t-j}y_{t-d}^3 + \beta_{\mathbf{4}j}y_{t-j}y_{t-d}^4) + \eta_t \quad (10)$$

and perform the following test hypothesis:

H_{0E}: $\beta_{2j}= \beta_{4j}=0$

H_{0L}; $\beta_{1j}= \beta_{3j}=0$

Their recommendation is to select the transition function according to the minimum p-value obtained from both null hypothesis. Nevertheless, it should be noted that neither test dominates one another in terms of power (van Dijk and Teräsvirta, 2000), and both are employed as complementary criteria.

2.3 Estimation of the ESTAR model

Once the transition variable (y_{t-d}) and the adequate transition function F(.) have been determined, the subsequent step consists on estimating the parameters in the ESTAR model. The estimation of the model is performed via Nonlinear Least Squares (Gallant and White, 1988) which is equivalent to quasi-maximum if the error terms are not normally distributed (van Dijk and Teräsvirta, 2000). The necessary and sufficient conditions for obtaining consistent and asymptotically normal estimators is that $u_t \sim i.i.d(0,\sigma^2)$ in addition to $\{y_t\}$ being a stationary and ergodic time series process (Klimko and Nelson, 1978; Tong, 1990).

2.4 Diagnostic check

The concluding stage in the estimations process entails conducting a series of misspecification tests to validate the fitted model. Aside from standard diagnostic tests, van Dijk and Teräsvirta (2000) point out that STAR model should also be subject to three misspecification tests. Firstly a test for serial independence in the residuals, where the test statistic can be considered a generalization of the LM-test for serial correlation model of Godfrey-Breusch-Pagan (1979). Secondly, a test of no remaining linearity, which is essential to establish whether or not the model has successfully captured the nonlinear dynamics of the process by incorporating an additional third regime into the alternative hypothesis. However, again under the null hypothesis of two regimes there is an identification problem which is circumvented by taking a higher order Taylor approximation (Eitrheim and Teräsvirta, 1996). Finally, an LM test for parameter constancy is performed based on the Taylor approximation used in the previous test to discriminate against other variants of the STAR family of models.

3. Empirical results

3.1. Data

The data employed are average monthly bilateral nominal exchange rate for the *Brazilian real* with respect to the euro, pound and dollar in indirect quotation. To proxy for the price level for each country's output, average consumer price indexes (CPI) are used, where Brazil is assigned as the foreign country in all cases. The data series are monthly observations ranging from December 1998 to June 2013. All the data has been extracted from the IMF's International Financial Statistic CD-Rom in the software Datastream and EUROSTAT website. The values have been transformed into natural logarithms. Deviations from PPP are then classified into two subgroups; restricted and unrestricted, depending on the nature of the restrictions imposed (whether or not both symmetry and proportionality are imposed). The empirical analysis regarding the testing and estimation of the STAR model are performed on the demeaned series of deviations from PPP as defined by these two cases.

3.2 Linear Cointegration tests

As preliminary analysis, a test of the general validity of the PPP hypothesis in its absolute form was carried out by testing both the hypotheses of proportionality (β_1=-β_2=1) and symmetry (β_1=-β_2). Both hypotheses were strongly rejected, repudiating the absolute version of PPP.[12] Rejection of the proportionality and symmetry suppositions stresses the need to work within general unrestricted trivariate models when it comes to testing for long-run cointegration. The reason being that if both conditions are imposed into the models, but these impositions are not reinforced by the data, the results will be biased towards finding no evidence of cointegration, and as such of mean-reversion.

[12]The data series are not seasonally adjusted, so that potential problems related to the distortion effect of seasonal adjustment to unit root tests are not taken into consideration (Ghusels, 1990).

The theory of cointegration requires the error terms from the linear combination of non-stationary variables to be stationary in order to support a long-run relationship between them.[13] Consequently, all nominal exchange rate series and price indices were each checked for a unit root by applying the Augmentd-Dickey-Fuller test (ADF), where the results are presented in Table 2.1. It is widely known that the ADF is extremely sensitive to the choice of lags included in the test. If the lag length is too small, then the remaining serial correlation in the errors will bias the test, whereas if it is too large, then the test will lose power against the alternative hypothesis . By employing the optimal lag length selection procedure proposed by Ng and Perron (1995), which has the advantage of resulting in minimal power lose, the optimal lag lengths for the standard unit root tests were selected.

From the results obtained in the standard ADF test for all the series examined individually, the null hypothesis of a unit root could not be rejected at the 1 percent significance level.[14] When a linear trend was introduced, in no case could the hypothesis of the series having a unit root be rejected. All these results suggest that the series are non-stationary (with the exception of the U.K. pound nominal exchange rate), which is consistent with previous stylized facts on nominal exchange rate series. Thereafter, the ADF test for the order of integration was performed on the first differenced series for which there was evidence of non-stationarity. In all cases, the hypothesis of a single unit root is rejected at all significance level, denoting that the series were indeed integrated of order one.

The results obtained from the Engle-Granger two-step procedure can be found on Table 2.2. Hitherto, the test involves examining the stationary properties of the residuals from the unrestricted cointegration regression. The choice of dependent variable as the normalisation variable - whether it be the nominal exchange rate, foreign or domestic prices - does not affect in

[13]The Shapiro-Wilk test was performed on each individual series to find whether the series are normally distributed. The results clearly reject the null of normality in all cases.

[14]The semi-parametric Phillip-Perron test was also carried out yielding analogous results, which are not included due to space constraint.

any of the cases the general cointegration results. The findings confirm the non stationarity of the residual in the case of all the real exchange rate series, at a 1 percent significance level; hence there is no evidence of cointegration between the nominal exchange rates and price differentials. Furthermore, the significance of the unit root hypothesis on the real exchange rate series was tested after having imposed the condition β_1=-β_2. Not surprisingly, the null of a unit root could not be rejected at any significance level. It can then be concluded that there is strong evidence contrary to the relative version of PPP, as indicated by the residual based cointegration analysis.

The Johansen test, which consists of a reduced rank regression technique that employs a Vector Auto Regression (VAR) framework, was performed. The lag length was chosen using the following joint criterion. Firstly, the lag length was observed from the multivariate Akaike information criterion (AIC) allowing for a maximum lag length of 12. The Schwarz information criterion was also taken into account, but generally disregarded as it tends to signal shorter lag lengths for the VAR system, which could result in serially correlated residual vectors. Secondly, given the optimal lag length, the estimated residual vectors from the system equation were tested for serial correlation using an LM test. In the presence of serial correlation, the lag length of the system was increased, and the process was repeated until the residuals were whitened. To account for measurement errors, a constant was included in the model following Cheung and Lai's (1993b) suggestion. The results from the Johansen trace and eigenvalue test, for the null hypothesis of at most r linearly independent cointegrating vectors are reported in Table 2.3. As shown, the null hypothesis of at most zero cointegrating vectors (r=0) cannot be rejected for a 5 percent significance level except for the unrestricted version of U.K. pound, which is rejected at 10 percent significance level[15]. Analogous to the previous cointegration test, the results provide no evidence of cointegration for the vector series given by e_t, P_t^* and P_t, thus failing to empirically

[15]The critical values for the trace and eigenvalue test statistic are tabulated work by Walter Enders, *Applied Economic Time Series*, *Wiley Series in Probability and Statistics*, 1995, page 420. For the finite sample properties of the Johansen test, see Osterwald-Lenum (1992).

support the PPP doctrine as a mean-reverting long-run relationship between nominal exchange rates and price differentials for the Brazilian case.[16]

It is necessary to point out, that failure to reject the hypothesis of no cointegration may have been attributed to the low power of the test in the presence of non-linearities in the data generating process.[17] The nonlinearity of the real exchange rate will not affect the unit root tests for the individual series, or the super-consistency of the OLS estimates of β_1 and β_2; however the results of the two step procedure will be considerably distorted. The power properties of the residual based cointegration test have been submitted to examination in several studies (Pippenger and Goering, 1993; Cheung and Lai, 1993a; van Dijk, 1999; Kapetanios, Shin and Snell, 2003). Using Monte Carlo simulation, the results highlighted the extremely low power of standard cointegration tests in rejecting the no cointegration null hypothesis, despite the presence of long-run equilibrium relationships between the cointegrating variables. As indicated by Michael et al. (1997), the incapacity of linear cointegration tests to find supportive evidence of PPP could be due to the existence of transaction costs that generate nonlinear behaviour, as in Dumas'(1992) model, rather than to the shortcomings of the hypothesis.

3.2. Test for Nonlinearity and Specification of STAR model

The results for the nonlinearity test via Teräsvirta's artificial regression (1994) are found in Table 2.1 based on the figures presented by the LM3 test statistic. To analyse the dynamic behaviour of deviations from the PPP hypothesis in a more rigorous manner, all the subsequent testing was performed for the univariate (restricted) and trivariate (unrestricted) forms of PPP. The results are

[16]The LR statistic for the cointegrating vector under H_0:β'=(1,1,-1) is rejected for the restricted model, once again invalidating absolute PPP.

[17]Alternatively, the low power of the statistical test of nonstationarity applied to real exchange may be attributed to the small sample employed during the current float. When taking into account expanded sample periods, evidence of cointegration is found for other set of exchange rates. For example, Grilli and Kaminksy (1991) by using a sample period that covers over one century, find evidence against the nonstationarity null hypothesis for the pound sterling-U.S. dollar real exchange rate series. The authors argue that the observed persistence of real exchange rates is more a function of the historical period covered, rather than the specific nominal exchange rate regime.

presented following the fore mentioned order, however for convenience these series will be referred to as real exchange rates from now on.

The length of the autoregression (p) for the real exchange rate was chosen on the basis of the PACF, conjunctly with information criterion and the Ljung-Box Portmanteau test to fully ensure the absence of serial correlation in the residuals. Neglected autocorrelation in the model will lead to false rejections of the linearity hypothesis in favour of nonlinear structure, so it was essential to guarantee the absence of serial correlation in the error terms of the linear representation of the series.

Hereafter, the optimal delay parameter(d) was determined according to the lowest p-value for the test hypothesis of nonlinearity, to ensure the strongest discrimination in favour of the correct specification of the transition function. The delay parameters by which the sequence of tests provided the strongest evidence are included in brackets. As explained in section 2.2, the F-test variant of the test should be taken more into consideration than its LM counterpart due to its size properties; in any case the p-values for both tests are reported. In addition, linearity tests in general are considerably sensitive to the presence of outliers, including the ones presented in this work, so the outlier-robust version of the test was also included (van Dijk et al., 1999; van Dijk and Teräsvirta, 2000). It has been shown that when the data generating process follows a linear model but the observations are contaminated by the presence of numerous outliers, then these tests usually tend to over reject the null hypothesis of linearity (van Dijk, Teräsvirta and Franses, 2001).[18]

From the results, for a ninety-five per cent significance level, the null hypothesis of linearity in the real exchange rate series are rejected except for the unrestricted Pound and Euro models. To strengthen the preliminary evidence that the series indeed reflect nonlinear behaviour, the Ljung-Box test for autocorrelation in the squared residuals was performed in conjunction with the Brock-

[18]Simulation results performed by van Dijk et al. (1999) suggest that the outlier robust LM-type test have good size properties in small samples. However, the downside to the robust version of the test is that in the absence of outliers its power properties reduce considerably. Visual inspection shows that use of the robust version of the test was warranted for the real exchange rate series studied.

Dechert-Scheinkman test for nonlinearity (Cromwell, Labys and Terraza, 1994).[19] All the results in Table 2.1 reinforce the evidence of nonlinear behaviour in the *real*'s real exchange rate series, excluding the fore mentioned cases. Altogether, the findings confirm the initial supposition that the linear cointegration tests performed in the previous subsection may have been weakened by the nonlinear nature of the adjustment process of the *real*. Therefore, failure to find a linear long-run relationship between nominal exchange rates and price indices does not invalidate the PPP hypothesis in a conclusive manner.

In accordance with Teräsvirta´s model selection procedure, the sequence of test hypothesis to determine the nature of the STAR model is presented in Table 2.2. Clearly, the LM statistic and the F-tests counterpart support an ESTAR representation in the case of the Euro and Pound, as the null hypotheses of H_{02}: $\beta'_{03}=0|\beta'_{04}=0$ are rejected with the smallest p-value. Likewise, Escribano and Jorda's model selection test statistics -both the LM and its F counterpart- point generally towards the same specification of the transition function. Strikingly, for the dollar both the unrestricted and restricted models fit an LSTAR specification with asymmetric dynamic behaviour. Given the mixed evidence presented by the LMH2 and LMHL statistics regarding the model selection criteria for the restricted model, the LSTAR version was opted for as it presented a better fit to the data.

The summary statistics and subsequent residual diagnostic of the estimated ESTAR models for the real exchange rate series are presented in Table 2.3. The speed of convergence was found to be relatively high albeit statistically insignificant at the 10 percent level (except for the Euro restricted model), suggesting that deviations from PPP equilibrium are somewhat not as persistent. However, it is difficult to obtain accurate estimates of the speed of the transition between the two regimes when this parameter is large as in the case of the unrestricted models (van Dijk et al., 2001). The reason being is that as γ becomes larger and larger, in the limit it becomes a TAR model were the transition function becomes a step function. Therefore, in order to obtain an accurate estimate of γ,

[19]The test statistics for the BDS tests and the Ljung-box test for squared residuals clearly rejected the null hypothesis of linearity for all the restricted models, and were not included in the tables due to space constraint.

it is necessary to have abundant observations in the neighbourhood of c*, since even very large changes in γ have only a minuscule effect on the transition function. For all these reasons, the estimates of γ may be rather imprecise and when judged by the t-statistic may seem insignificant (Bates and Watts, 1988). In any case, there is strong evidence in favour of nonlinear mean reversion in all the real exchange rates considered, as overall the models are globally stable with $\sum_{j}^{p} = 1\left(\pi_j + \pi_j^*\right) < 1$.

A thorough evaluation of the adequacy of the estimated two regime ESTAR models was performed through a series of diagnostic tests. The sequence of misspecification tests ranging from the more standard hypothesis tests of no residuals autocorrelation, normality and ARCH effects, to the more explicit STAR misspecification tests are displayed in Table 2.4. In this framework, the test of no remaining nonlinearity and parameter constancy which employ the same artificial regression, can be used as a test against a multiple regime STAR (MSTAR) model alternative (for example a Time Varying Smooth Transition Autoregressive Model; TV-STAR). The generalization of these tests to other specific variants of the STAR family of models can be obtained in the work of van Dijk, Teräsvirta and Franses (2001, pp.16-18).

The Ljung-Box test statistic is unable to reject the null hypothesis of non-autocorrelated errors for the models considered at a 5 percent level, which indicate that the residuals follow a white-noise process. In all cases, the Jarque-Bera's normality test highlights the fact that the series are non-normally distributed as expected, and there is some significant level of kurtosis and skewness in them. This is somewhat not surprising as the *real* has been following an appreciating trend for the past decade, which ultimately affects its distribution.

The three main tests in relation to the correct specification of the STAR two regime model show satisfactory results in general, and validate the choice of the STAR specification. The real exchange rate series seems to be correctly depicted by a two regime model as the null of no remaining nonlinearity cannot be rejected at any level by the LM.RNL3 test statistic. Furthermore, the test for serial independence proposed by Eitrheim and Teräsvirta (1996), LM-SI(q), presents clear evidence of no serial dependence for lags 6 and 12, providing clear indication that the model is correctly specified. Conclusively, the parameters of the models appear generally to be stable in the time frame considered, dismissing the specification requirement of a time varying model. This is not the case for the Pound, where the null is rejected at 5 percent level, which indicates that perhaps a time-varying variant of the STAR family of models should be adopted.

All in all, the evidence obtained supports the adequacy of the STAR models in providing an acceptable representation for the adjustment process towards PPP equilibrium for real exchange rate series. Moreover, based on the estimates of γ, there exists a relatively low level of persistence in the deviations from PPP (however as pointed out before, these estimates should be taken with caution), and convergence is certainly present. From Graph 1, which depict the estimated transition functions, it is possible to observe how the concentrated values of the transition function indicate a moderately high speed of convergence, and how the series are generally found away from their equilibrium values.

3.3 Dynamic Behaviour of the Real Exchange Rate

In order to analyze the dynamic characteristic of the estimated models, the inverted roots are computed for each individual model. The middle and outer regimes considered are given by equations (4) and (5) respectively. The results are presented in Table 2.5 which displays the most prominent inverted roots for each model and regime and portrays the dynamic behaviour of the estimated models.

Interestingly, in the case of the euro ESTAR model, both regimes are explosive as their modulus are greater than one, implying that the real exchange rate changes movement to and from equilibrium level erratically and at a high speed, although the series is globally stable. The estimated ESTAR model for the pound displays a somewhat asymmetric behaviour between the outer and middle regime, since the later has an explosive root which means that the series quickly explodes through the threshold value. However the outer regime is stable and the exchange rate converges relatively rapidly to parity.

For the restricted LSTAR dollar model, both regimes are stable which indicates that the real exchange rate tends to move towards parity even for small deviations. This is intuitively consistent with the fact that transport costs may be lower in relation to the U.S. which implies a narrower band of fluctuation around PPP equilibrium. Similarly, the unrestricted version of the dollar displays an explosive outer regime and a stable inner one. This suggest that the real exchange rate series will tend to stay around equilibrium value for small deviations, and will manifest rapid mean-reverting behaviour for considerable ones.

A simple out of sample forecast for twenty periods ahead was also performed to highlight the nonlinear mean reverting properties of the real exchange rate series. From Graph 2, one can observe that the real exchange rates tend to converge to the threshold value, thus displaying strong non-linear mean-reverting properties. Strikingly, as Figure 4 shows, it is expected for the pound to follow a depreciating trend in the following months, only to correct itself around the year 2013-2014, stressing the nonlinear dynamic properties of the real exchange rate series.

4.Conclusions

This work attempts to model the dynamics of the *real*'s real exchange rate adjustment process to long-run purchasing power parity relative to a number of currencies by applying a non-linear framework, in order to reconcile the mixed empirical evidence with the general Casselian view. Strong empirical evidence was found on the *real*'s real exchange rate being appropriately characterized by a non-linear STAR model, exhibiting mean-reverting properties as well as accounting for the moderate persistence observed in the data. Moreover, there was strong evidence found of non-linearities for the real exchange rate series, which can conceivably explain the empirical failure to find a cointegrating relationship between nominal exchange rates and cross-country price differentials. All in all, this signifies that there is still hope for the PPP doctrine as a valid equilibrium condition in exchange rate determination.

The linear cointegration tests performed for all three nominal exchange rates failed to support any equilibrium long-run relationship between the cointegrating variables. However, the linearity test based on Teräsvirta's artificial regression generally rejected the null hypothesis of linearity for the real exchange rate series, suggesting that the failure to find cointegration may be attributed to the low power of the tests in the presence of non-linearities in the data generating process. This fact highlights the need to include non-linearities in standard residual based cointegration tests. The real exchange rate series were then classified into LSTAR and ESTAR models, and the estimated models provided satisfactory residual diagnostics in a broad sense. All the indications point out that the specified models depict a satisfactory description of the dynamic process of the real exchange rate series, as described in Dumas' (1992) model. In fact, the evidence highlights that small deviations from PPP are non-stationary and persistent, whilst large deviations exhibit rapid mean-reverting behaviour, findings that are recurrent in similar works. This implies that the models are globally stable, supporting the PPP hypothesis for the Brazilian *real*'s case.

5. Appendix

Table 1.1 - Augmented Dickey-Fuller Test

Country	Tau statistic	et	pt	pt*	Δet	Δpt	Δpt*
Euro Area	τ_t	-1.599	-1.4481		-12.118*	-3.654**	
		(1)	(14)		(0)	(13)	
U.S.	τ_t	-1.733	-1.826		-9.709*	-5.662*	
		(2)	(12)		(1)	(11)	
U.K.	τ_t	-4.435*	-1.851		-	-3.241**	
		(4)	(13)			(12)	
Brazil	τ_t			-1.989			-5.170 *
(Foreign Currency)				(14)			(13)

Note: a. * Indicates the unit-root null hypothesis is rejected at the 1% significance level, where the critical values are obtained from the response surface estimates given in MacKinnon (2010).

** indicates at 5% and *** at 10%.
b. The numbers in parenthesis are the lag order included in the ADF test. The lag order selection criteria was chosen as suggested by Ng and Perron (1993).
c. The critical values for the ADF test without a trend for 1%, 5% and 10% critical level are: -3.962, -3.411 and -3.128.
d. The ADF test included a trend except for et, therefore a different set of critical values are required. The 1%, 5% and 10% critical values are respectively: -4.018, -3.441 and -3.141.

Table 1.2. The Engle and Granger two-step Test

a. Unrestricted model

step 1. Regress $e_t = c + \beta_1 P_t + \beta_2 P_{t*} + y_t$

	Euro Area	U.S.	U.K.
c	22.689	18.746	0.122
β1	-11.586	-9.415	-1.427
β2	4.073	4.069	1.1

step 2. Regress $\Delta y_t = \sigma y_{t-1} + \sum \mu \Delta y_{t-p} + \varepsilon_t$

	Euro		U.S.		U.K.	
σ	-0.017	(0.010)	-0.022	(0.013)	-0.017	(0.010)
ADF	-2.405		-1.674		-1.566	
	[p=6]		[p=2]		[p=2]	

b. Restricted model (symmetry imposed)

step 1. Regress $e_t = c + \beta_1 P_t + \beta_2 P_{t*} + y_t$

	Euro	U.S.	U.K.
c	-0.590	-1.611	-1.455
β1	-0.481	-0.912	-1.046
β2	0.481	0.912	1.046

step 2. Regress $\Delta y_t = \sigma y_{t-1} + \sum \mu \Delta y_{t-p} + \varepsilon_t$

	Euro		U.S.		U.K.	
σ	-0.027	(0.015)	-0.014	(0.009)	-0.014	(0.009)
ADF	-1.796		-1.515		-1.405	
	[p=1]		[p=2]		[p=2]	

Note: a. The number in parenthesis denotes the standard errors for the lagged value of the residual.

Squared parenthesis refer to lag order.

b.The critical value of the ADF test for a cointegrating regression containing an intercept for T=200 are 3.51, 3.78 and 4.34 for 10%, 5% and 1% significance levels respectively.

These critical values were calculated using Monte Carlo methods by Robert F. Engle and Byung Sam Yoo and

obtained from their paper ¨Forecasting and Testing in Co-Integrated Systems," Journal of Econometrics, Vol. 35, 1987, page157.

Table 1.3 - Johansen Cointegration test

a. Test for Cointegration Unrestricted Model (n = 3)									
Country	**λMax**			**Trace**			**β'**	**H0: β'=(1,1,-1)**	**p**
	H0: r = 0	H0: r ≤ 1	H0: r ≤ 2	H0: r = 0	H0: r ≤ 1	H0: r ≤ 2			
Euro Area	15.888	8.260	2.141	26.290	10.402	2.141	(1, 20.477 -7.092)	7.621 (0.006)	14
U.S.	20.531	2.622	1.869	25.023	4.491	1.869	(1, 46.772,-19.423)	13.73 (0.00)	3
U.K.	24.089	7.846	2.23	34.166	10.077	2.231	(1,9.477,-5.226)	12.97 (0.0)	14

b. Test for cointegration Bivariate Model (n = 2)							
Country	**λMax**		**Trace**		**β'**	**H0: β'=(1,1)**	**p**
	H0: r = 0	H0: r ≤ 1	H0: r = 0	H0: r ≤ 1			
Euro Area	6.683	0.725	7.408	0.725	(1, 0.076)	2.33 (0.127)	14
U.S.	6.962	1.264	8.227	1.264	(1, -1.662)	4.479 (0.034)	5
U.K.	8.8439	0.0572	40.114	0.0572	(1, 11.188)	34.66 (0.0)	5

Notes: a. Critical values for the likelihood ratio statistic are based on the simulated values tabulated in Johansen and Juselius(1990). Where "n" denotes the number of variables in the system and "r" is the cointegration rank.

b.The λmax statistic has the following critical values at the respective significance levels. For n -r = 1; 7.563(10%), 9.094(5%) and 12.740(1%), for n - r = 2; 13.781(10%), 15.752(5%) and 19.834(1%), for n - r = 3; 19.796(10%), 21.894(5%) and 26.409(1%).

c.The Trace statistic has the following critical values at the respective significance levels. For n -r = 1; 7.563 (10%), 9.094(5%) and 12.741(1%), for n - r = 2; 17.957 (10%), 20.168(5%) and 24.988(1%), for n - r = 3; 32.093(10%), 35.068(5%) and 40.198(1%).

d. The cointegrating vector β' has been normalized with respect to the nominal exchange rate variable et. The LR test for the indentifying restriction H0: β' = (1, 1, -1) has a X^2 under the null with 2 degrees of freedo. The marginal significance levels are presented in brackets.

e. p indicates the order of the the Vector Autoregressive model.

Table 2.1 - Tests for linearity of real exchange rate series

Model	optimal d	lag order	p-value	(p-value outlier robust)
Euro				
Restricted	3	2	0.000	(0.0001)
Unrestricted	8	3	0.133	(0.1865)
Dollar				
Restricted	3	2	0.0020	0.003
Unrestricted	5	3	0.017	0.033
Pound				
Restricted	3	2	0.004	(0.006)
Unrestricted	4	2	0.142	0.262

note: a. The optimal d was chosen on the basis of the minimum p-value of the linearity over the range {1-8}, for a given value of the order of the autoregression

b. The p-value corresponds to the LM3 statistic for the linearity hypothesis H_{0L} presented in section 2.2

c. We cannot reject the null hypothesis of linearity for the Pound/Unrestricted and Euro/Unrestricted model.

Table 2.2 - Teräsvirta's Model Selection procedure/Escribano-Jorda's Procedure

Model	Standard Test				Outlier Robust Test				Selected Model
	Chi-squared variant		F-variant		Chi-squared variant		F-variant		
Euro									
Univariate									*ESTAR*
LMH1	12.83	(0.005)	4.46	(0.005)	10.38	(0.0155)	3.55	(0.016)	
LMH2	13.23	(0.004)*	4.53	(0.004)*	14.07	(0.0028)*	4.84	(0.003)*	
LMH3	8.36	(0.039)	2.73	(0.046)	7.23	(0.0648)	2.34	(0.075)	
LMHE	11.08	(0.086)*	1.80	(0.102)	10.10	(0.1206)	1.63	(0.142)	
LMHL	8.61	(0.197)	1.38	(0.226)	6.76	(0.3433)	1.07	(0.382)	
Trivariate	-	-	-	-	-	-	-	-	*LINEAR*
Dollar									
Univariate									*LSTAR*
LMH1	5.97	(0.113)	1.99	(0.117)	1.93	(0.586)	0.63	(0.597)	
LMH2	16.49	(0.0008)*	5.76	(0.0009)*	19.63	(0.0002)*	7.00	(0.0001)*	
LMH3	3.04	(0.386)	0.96	(0.414)	2.85	(0.416)	0.90	(0.444)	
LMHE	9.39	(0.153)	1.51	(0.178)	7.94	(0.242)	1.27	(0.276)	
LMHL	14.38	(0.026)*	2.39	(0.031)*	13.40	(0.037)*	2.21	(0.045)*	
Trivariate									*LSTAR*
LMH1	13.37	(0.009)*	3.46	(0.009)*	12.20	(0.016)*	3.13	(0.016)*	
LMH2	7.55	(0.109)	1.84	(0.125)	7.34	(0.119)	1.78	(0.135)	
LMH3	4.65	(0.325)	1.08	(0.367)	3.99	(0.408)	0.92	(0.451)	
LMHE	10.66	(0.222)	1.25	(0.272)	9.67	(0.289)	1.13	(0.346)	
LMHL	5.65	(0.686)	0.65	(0.739)	5.03	(0.754)	0.57	(0.799)	

Table 2.2 - continued

Pound									
Univariate									*ESTAR*
LMH1	3.68	(0.298)	1.21	(0.308)	0.75	(0.861)	0.24	(0.866)	
LMH2	12.01	(0.007)*	4.08	(0.008)*	13.09	(0.004)*	4.47	(0.005)*	
LMH3	8.82	(0.032)	2.88	(0.038)	8.10	(0.044)	2.63	(0.052)	
LMHE	12.46	(0.053)	2.04	(0.063)	10.89	(0.092)	1.77	(0.109)	
LMHL	16.92	(0.009)*	2.86	(0.011)*	15.24	(0.018)*	2.54	(0.022)*	
Trivariate	-	-	-	-	-	-	-	-	*LINEAR*

note: a. The test statistics correspond to the following null hypotheisis in section 2 : H_{01}, H_{02}, H_{03}, H_{oHe}, H_{oHL}

b. p-values are in brackets. The* stands for the smallest p-value associated with each test statistic.

Table 2.3 - Estimates of STAR models

Euro Restricted:ESTAR

$$y_t = -\underset{(0.1656)}{0.4456} + \underset{(0.3993)}{2.5648}\, y_{t-1} - \underset{(0.7574)}{2.698}\, y_{t-2} + \left(\underset{(0.1657)}{0.4479} - \underset{(0.4099)}{1.366}\, y_{t-1} + \underset{(0.7620)}{2.494}\, y_{t-2} \right)$$

$$\times \left[1 - \exp\left\{ -\underset{(6.7518)}{18.121} \left(1/\sigma^2\right) \left(y_{t-3} + \underset{(0.0127)}{0.3969} \right)^2 \right\} \right]$$

$S^2 = 0.00137,$

$D-W = 0.229, Q(6) = 0.989, Q(12) = 0.668, JB = 61.370, ARCH(1) = 0.497, ARCH(4) = 0.347$

Dollar Restricted:LSTAR

$$y_t = -\underset{(0.2379)}{0.3936} + \underset{(0.3861)}{0.386}\, y_{t-1} - \underset{(0.3750)}{0.24}\, y_{t-2} + \left(\underset{(0.2379)}{0.3995} + \underset{(0.3961)}{0.999}\, y_{t-1} - \underset{(0.3876)}{0.192}\, y_{t-2} \right)$$

$$\times \left[1 + \exp\left\{ -\underset{(4.4014)}{6.6253} \left(1/\sigma\right) \left(y_{t-3} + \underset{(0.0407)}{0.4449} \right) \right\} \right]^{-1}$$

$S^2 = 0.00129$

$D-W = 0.581, Q(6) = 0.665, Q(12) = 0.826, JB = 30.330, ARCH(1) = 0.598, ARCH(4) = 0.409$

Dollar Unrestricted: LSTAR

$$y_t = \underset{(0.00038)}{0.0068} + \underset{(0.081)}{1.2716}\, y_{t-1} - \underset{(0.1283)}{0.3839}\, y_{t-2} + \underset{(0.0848)}{0.1034}\, y_{t-3} + \left(-\underset{(0.0507)}{0.111} - \underset{(0.3500)}{0.0239}\, y_{t-1} - \underset{(0.5375)}{0.9899}\, y_{t-2} + \underset{(0.4172)}{0.982}\, y_{t-3} \right)$$

$$\times \left[1 + \exp\left\{ -\underset{(2.3737)}{3.294} \left(1/\sigma\right) \left(y_{t-5} - \underset{(0.0391)}{0.2825} \right) \right\} \right]^{-1}$$

$S^2 = 0.0016$

$D-W = 0.630, Q(6) = 0.710, Q(12) = 0.939, JB = 15.610, ARCH(1) = 0.834, ARCH(4) = 0.766$

Table 2.3 (continued)

Pound Restricted: ESTAR

$$y_t = -\underset{(0.0228)}{0.0416} + \underset{(0.6513)}{1.947}\, y_{t-1} + \underset{(0.9141)}{0.439}\, y_{t-2} + \left(\underset{(0.0231)}{0.04344} - \underset{(0.6560)}{0.7287}\, yt - 1 - \underset{(0.9183)}{0.6778}\, y_{t-2} \right)$$

$$\times \left[1 - \exp\left\{ -\underset{(15.5416)}{15.938} \left(y_{t-3} - \underset{(0.0061)}{0.031} \right)^2 \right\} \right]$$

$$S^2 = 0.00129$$

$$D-W = 0.1096, Q(6) = 0.728, Q(12) = 0.904, JB = 53, ARCH(1) = 0.137, ARCH(4) = 0.487$$

Note. a. The values in parenthesis are standard errors.

b. D-W stands for the p-value of the Durbin-Watson test for the presence of autocorrelation in the residuals

c. Q(6) and Q(12) stand for the p-values of the Ljung-Box statistic for residual autocorrelation for lags 6 and 12 respectively.

c. ARCH(1) and ARCH(4) stand for the p-values for Engle's LM test for ARCH effects with lags 1 and 4 respectively

d. JB stands for the test statistic in Jarque-Bera's normality test

Table 2.4 - Residual Diagnostics

	M Tests for No Remaining Nonlinearity		LM Tests for Serial Independence		LM Tests for Parameter Constancy	
Model	**LM.RNL**	**p-val(X^2.)**	**LM-SI(6) LM-SI(12)**	**p-val(X^2.)**	**LM.C3**	**p-val(X^2.)**
Euro						
Univariate	31.390	0.142	2.679 13.422	0.847 0.339	37.810	0.036
Trivariate	-	-	-	-	-	-
Dollar						
Univariate	26.899	0.309	6.548 12.9013	0.365 0.3762	47.663	0.003
Trivariate	45.1384	0.0374	7.0068 11.2066	0.3202 0.5112	47.355	0.0229
Pound						
Univariate	26.95	0.306	4.906 9.011	0.555 0.701	41.433	0.014
Trivariate	-	-	-	-	-	-

Note:

a. The LMSI(q) statistic is asymptotically X^2 distributed with 1 degree of freedom

b. The test statistic LM.RNL has an asymptotic X^2 distribution with 3(p+1) degrees of freedom

c. The test statistic LM.C3 has the same asymptotic distribution as the LM.RNL3 statistic.

d. For a more exhaustive review see van Dijk and Teräsvirta (2000).

Table 2.5 - Most Prominent Inverted Roots

Model	Regime	Most prominent Inverted roots	Modulus
Euro			
Restricted	*Middle*	1.282±1.026	1.643
	Outer	1.038	1.038
Unrestricted	*Middle*	-	-
	Outer	-	-
Dollar			
Restricted	*Middle*	0.1932±0.4505	0.4902
	Outer	0.2598	0.2598
Unrestricted	*Middle*	0.1412± 0.2909	0.3234
	Outer	-0.3485±1.157	1.2079
Pound			
Restricted	*Middle*	2.1482	2.1482
	Outer	-0.3632±0.7372	0.8218
Unrestricted	*Middle*	-	-
	Outer	-	-

Graph 1. Estimated Transition Functions.

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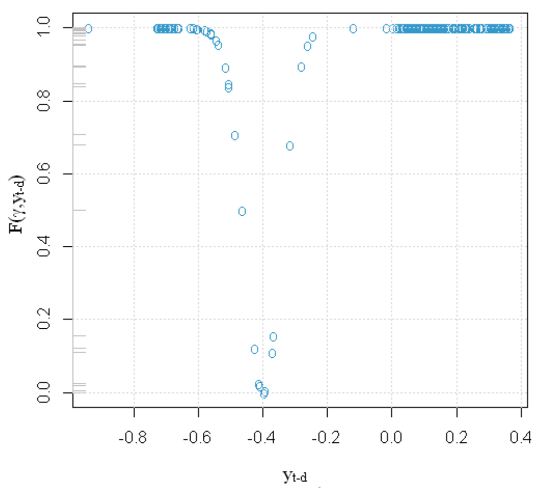
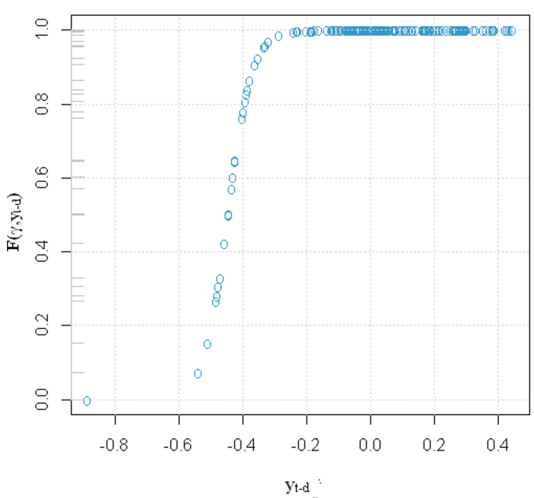
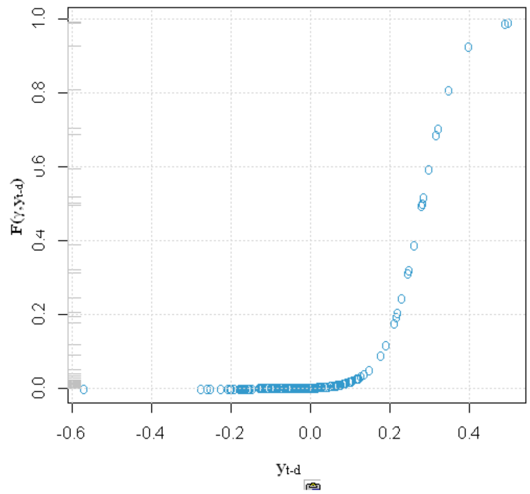
a. Euro Restricted model



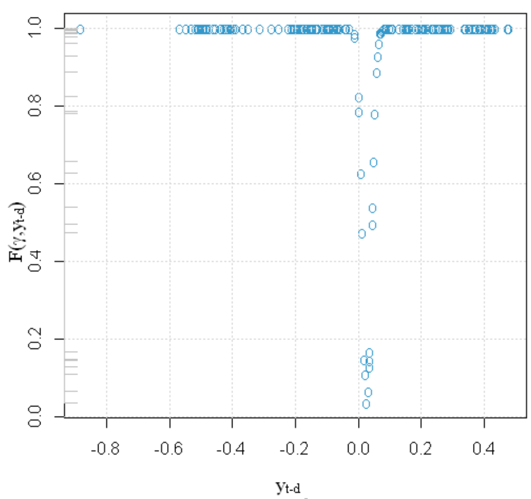
b. Dollar Restricted model



c. Dollar Unrestricted Model



d. Pound Restricted Model



Graph 2. Forecast.

a. Euro Restricted Model

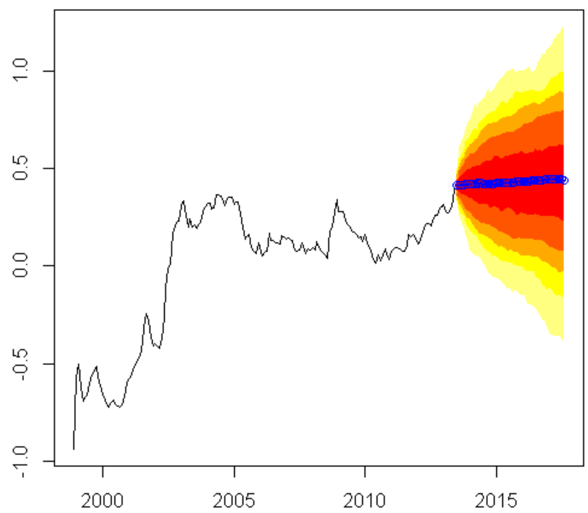


Fig1. Euro (log) real exchange rate forecast

b. Dollar Restricted model

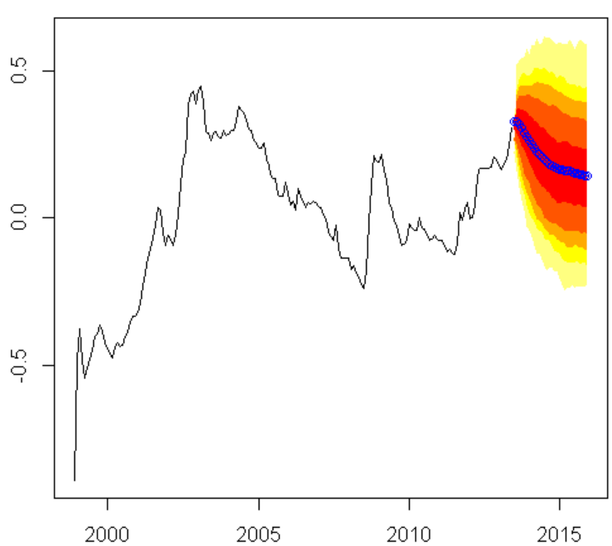


Fig2. Dollar (log) real exchange rate forecast

c. Dollar Unrestricted Model

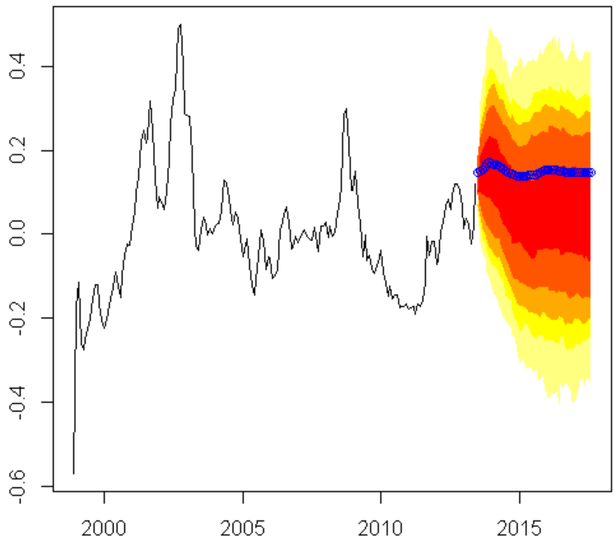


Fig3. Dollar (log) real exchange rate forecast

d. Pound Restricted Model

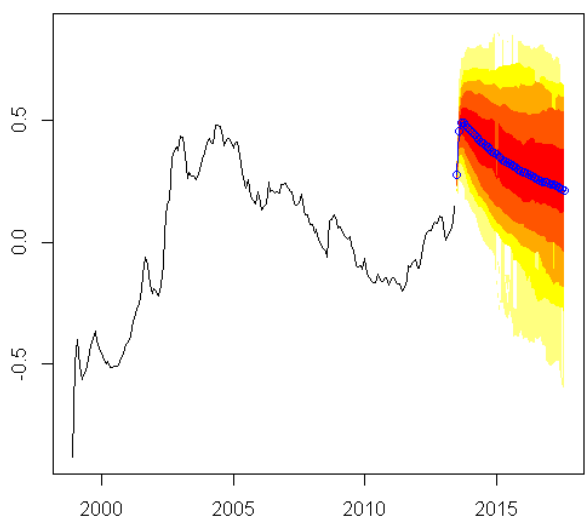


Fig4. Pound (log) real exchange rate forecast

Notes. a. The forecast was performed using parametric bootstrap in the "RSTAR" package for R, for 20 periods ahead. b. The X-axis represent the time period, whereas the Y-axis refer to the (log) real exchange rate.

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