THE PRIVATE VALUE OF HAVING ACCESS TO DERIVATIVE SECURITIES: AN EXAMPLE USING COMMODITY OPTIONS

DAVID W. BULLOCK and DERMOT J. HAYES

Abstract

Antonovitz and Roe (1986) developed a Bayesian framework for the construction of a money metric measure of the value of information. This paper extends the Antonovitz-Roe framework to the construction of a money metric measuring an investor's private value for having access to a derivative security market, given a constant information set containing moments of the underlying price distribution. In particular, the case of commodity options is examined. Results from the analytical model show that the value measure can never be negative and is equal to zero only when the investor's expected marginal speculative return to the option is equal to the hedge-adjusted marginal expected speculative return to the underlying futures contract. Simulation analysis of the analytical model indicates that information on the mean of the underlying price distribution is more significant than information on the variance in affecting the access value of the option.

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I. Introduction

The definition of a security, in the widest sense, is an exchangeable contract that grants a claim(s) to a contingent commodity(s). Securities markets allow investors to manage risk, allocate wealth over time, and earn returns to information collection. A derivative security is a security whose final payoff is totally determined by the payoff on an underlying physical or security. An example of a derivative security is a commodity option, whose final payoff is completely determined by the price of an underlying futures contract.

For an economic agent facing uncertainty, private information on the moments of the uncertain variable’s probability distribution can have intrinsic value in a Bayesian sense (Antonovitz and Roe, 1986). It is important to discriminate between whether the agent’s information is public or private. Hirshleifer (1971) provided an example where the public provision of information makes some agents worse off.

The gain in individual utility from the availability of private information is constrained by the consumption, production, and investment opportunities sets of the individual economic agent. By application of the LeChatelier Principle, it is evident that the expansion of any of these opportunity sets will never lower the utility of the economic agent.1

The introduction of a derivative security into an investor’s portfolio expands the investment opportunities set of the investor (Cox and Rubinstein, 1985) if the pattern of returns for the derivative security cannot be replicated by combinations of existing securities.2 The addition of such a derivative security to an investor’s portfolio may increase the private value of the investor’s information set since the information may be used more efficiently. This improvement is realized through the expanded set of choices in the investor’s investment opportunities set.

This paper addresses the question of how much a typical investor is willing to pay to have access to a unique derivative market. In particular, the commodity options market will be examined.

In the next section, a procedure for constructing a money-metric measure of access value is illustrated. In particular, the measure is constructed by using a certainty equivalent representation of the theoretical model of Antonovitz and Roe. The procedure assumes that the investor’s utility of wealth satisfies the von Neumann-Morgenstern axioms of expected utility theory.

The portfolio models with and without commodity options are derived in Section III. By assuming a constant absolute risk aversion (CARA) utility function, the money-metric measure of access value is constructed using the procedure outlined in the previous section. Using the analytical representation, it is shown that the access value of an option is zero only when the expected marginal speculative returns to the option are equal to the hedge-adjusted marginal speculative returns to the underlying futures. Otherwise, access value increases exponentially with differences in these terms.

More complex comparative statics results are examined in Section IV using a stochastic simulation of the portfolio model. In particular, the analysis examines the relative influence of moment information on the access value of the option using regression analysis on the
simulation results. The regression results indicate that information on the future spot price mean is significant in increasing the access value of option. The results for the future spot price variance were mixed, both in significance and direction, and indicate the possibility of a tradeoff between a risk and return effect for the variance. The final section (Section V) provides a summary of the results of the previous sections.

II. Construction of the Money-Metric

Consider an investor who has access to a set $\Omega$ of information. Suppose the investor has a von Neumann-Morgenstern utility function $U(\cdot)$ of profits $\pi$ from an investment portfolio. The instrument set of the investor’s current portfolio $\gamma_o$ does not contain the derivative asset $z$. The investor has the opportunity to purchase an enriched portfolio $\gamma_b = \{\gamma_o, z\}$. If $\theta_i$ represents the set of possible return structures for portfolio $\gamma$, then $z$ is a unique asset if $\theta_a \subset \theta_b$ and $\theta_b \not\subset \theta_a$. Following Antonovitz and Roe, a money-metric for access value (AV) can be constructed as

$$E[U(\pi_b^* | \Omega)] = E[U(\pi_o^* + AV) | \Omega],$$

where $E[\cdot]$ is the expectation operator, and $\pi_o^*$ is equal to profits from holding the optimal portfolio containing instrument set $\gamma_o$.

A certainty equivalent is defined as the risk-free level of income that is equivalent in expected utility to a risky lottery. Using certainty equivalents, equation (1) can be rewritten as

$$CE^*_b = CE^*_a + AV,$$

or

$$AV = CE^*_b - CE^*_a,$$

where $CE^*_a$ is the optimized certainty equivalent for portfolio $\gamma_a$.

Suppose the investor’s preference for portfolio profits can be represented by the following constant absolute risk aversion (CARA) utility function:

$$U(\pi) = -\exp(-A\pi),$$

where $A$ is the Arrow-Pratt coefficient of absolute risk aversion. The certainty equivalent of (2) can be represented by the following Taylor series expansion:

$$CE = E[\pi] - \frac{1}{2} A E[s^2] + \frac{1}{6} A^2 E[s^3] - \frac{1}{24} A^3 E[s^4] + \ldots,$$

where $s = \pi - E[\pi]$.

For analytic purposes, it is important to determine at what point to cut off the Taylor series as an approximation to the certainty equivalent. Models that consider the first two terms of the expansion in equation (3) are able to provide less ambiguous results. Such two-parameter certainty equivalent models are widely used in the finance and economics literature.
The third-order term in equation (3) represents the skewness preference of the investor $A^2/6$ multiplied by a measure of skewness $E[s^3]$ on the underlying profit distribution. This term is positive (negative) if the distribution of wealth is skewed to the right (left). Most certainty equivalent models completely ignore terms above the third order because these terms are generally quite small in magnitude as compared to the mean.

In this paper, the two-parameter model is assumed to be the appropriate model if at least one of the following two conditions is met:

(C1). The investor’s perceived distribution of wealth is symmetric (i.e., $E[s^3] = 0$), or

(C2). The ratio $\frac{1}{6}A^2E[s^3]/E[\pi]$, which is the skewness term over the mean in equation (3), is sufficiently small to be ignored.

Note that satisfaction of (C1) implies satisfaction of (C2), but not vice versa. This suggests a two-step procedure for determining the appropriateness of the two-parameter model. The first step involves analytically deriving the probability distribution of wealth. If this distribution is symmetric, then condition (C1) is satisfied and the two-parameter model can be used. If condition (C1) is not satisfied, condition (C2) is tested by empirically deriving sample values of the skewness ratio using repeated trials of a simulation model. A two-tail $t$-test is conducted on the ratio samples to test the null hypothesis that the true population mean of the ratio is zero. If the null hypothesis is rejected, with probability of type I error equal to 0.10, then the three-parameter model is used. Otherwise, the two-parameter model is used. This two-step test is a variant of the approximation tests employed in Levy and Markowitz (1979) for nonnormal security returns, and in Hanson and Ladd (1991) for truncated distributions.

If either condition (C1) or (C2) is satisfied, equation (3) can be rewritten as

$$CE = E[\pi] - \frac{1}{2}A \cdot Var[\pi],$$

where $Var[\cdot]$ is the variance operator. The access value for the derivative asset can be derived as

$$AV = m - \frac{1}{2}A \cdot v,$$

where $m = E[\pi^*_0 | \Omega] - E[\pi^*_0 | \Omega]$ and $v = Var[\pi^*_0 | \Omega] - Var[\pi^*_0 | \Omega]$.

### III. The Portfolio Models

Consider a two-period model in which a commodity investor determines portfolio positions in an initial time period (period zero) by maximizing the expected utility of future (period one) profits from holding the portfolio. Assume that the positions must be placed in period
zero and can only be offset in period one. Also, assume that the investor has a CARA utility function represented by equation (2).

There are two portfolios that are considered in this model. Portfolio $\gamma_a$ contains inventory of a storable agricultural commodity $I$ and a futures contract $X$ for delivery of the physical commodity in period 1. Portfolio $\gamma_a$ is equal to $\gamma_a$ plus a single put option contract $R$ on the underlying futures. Both long and short positions can be held in $I$, $X$, and $R$. The futures (inventory) position is long if $X > 0$ and short if $X < 0$. The put option position is short if $R > 0$ and long if $R < 0$.

The following assumptions are made for simplification purposes:

(A1). The holding cost of physical inventory can be represented by a quadratic function.
(A2). Transactions costs for the futures and put option are zero.
(A3). The par delivery market on the futures contract is identical to the spot market for the physical inventory.
(A4). The option expires in period 1.

Since futures can only be offset at expiration, and given assumption (A3), implies that the period one spot and futures prices will be identical (i.e., no basis risk). The addition of assumption (A4) implies that the option on the futures is identical to an option on the physical commodity. Also note that a call option is redundant in portfolio $\gamma_1$ since it can be replicated by a "synthetic call" using the put option and futures.

For $\gamma_a$, the investor’s period one profit function can be represented as

$$\pi_a = (\bar{P}_1 - P_0)I_a - \frac{1}{2} g \bar{P}_a^2 + (\bar{P}_1 - P_f)X_a,$$

where $\bar{P}_1$ is the spot market price in period one (unknown in period zero), $P_0$ is the spot market price in period zero, $P_f$ is the futures price in period zero, and $g$ is the quadratic holding cost coefficient. The $a$ subscript on the portfolio instruments indicates that the optimized demands for these instruments correspond to portfolio $\gamma_a$. The investor’s optimization problem can be stated as follows:

$$\max_{I_a, X_a} EU(\pi_a) = E[-\exp(-A\pi_a)].$$

Assume that the investor believes that the probability distribution for $\bar{P}_1$ is normal and the investor’s information set contains the moments for this distribution $\Omega = [\mu_P, \sigma_P^2]$. Note that $\pi_a$ is normally distributed because it is linear in $\bar{P}_1$. Thus, condition (CI) is satisfied and the certainty equivalent of $\pi_a$ can be written as

$$CE_a = E[\pi_a | \Omega] - \frac{1}{2} A \text{Var}[\pi_a | \Omega],$$

where
Substituting equations (7) and (8) into (6) and maximizing with respect to \( I_a \) and \( X_a \) gives the following portfolio equations:

\[
I_a^* = \frac{P_f - P_0}{g},
\]

\[
X_a^* = \frac{\mu_p - P_f}{A\sigma_p^2} - I_a^*.
\]

Equations (9) and (10) give the standard results for a two-parameter model containing spot and futures markets with no basis risk. Inventory demand is a function of the expected marginal returns to storage, \( P_f - P_0 \), divided by the second derivative of the storage cost function \( g \). Note that the futures price \( P_f \) is substituted for the forecast of the future spot price \( \tilde{P}_1 \) in determining the optimal inventory level. The investor is able to perfectly hedge the inventory position in the absence of basis risk. Therefore, the futures contract is equivalent to a forward contract. This is consistent with the mean-variance results of Holthausen (1979) and the expected utility results of Feder, Just, and Schmitz (1980).

The optimal futures position is the sum of a speculative component \( (\mu_p - P_f)/A\sigma_p^2 \) and a hedging component \(-I_a^*\). The speculative component is equal to the ratio of the marginal expected speculative return to the futures position over a risk premium. Note that the hedge ratio between the futures position and the inventory position is equal to minus one because of the absence of basis risk.

For \( \gamma_b \), the investor's period one profit function can be represented as

\[
\pi_b = (\tilde{P}_1 - P_0)I_b - \frac{1}{2} g J_b^2 + (\tilde{P}_1 - P_f)X_b + [P_R - \text{Max}(0, K - \bar{P}_1)] R_b,
\]

where \( P_R \) is the put option premium in period zero, and \( K \) is the strike price on the put option. The \( b \) subscript on the portfolio demands is used to indicate that the optimized demand functions correspond to portfolio \( \gamma_b \). The investor's optimization problem can be stated as follows:

\[
\max_{I_b, X_b, R_b} EU(\pi_b) = E[-\exp(A\pi_b)].
\]

Assume that the investor has the same preferences and information set (\( \Omega \)) as for portfolio \( \gamma_a \). We cannot assume that \( \pi_b \) is distributed normally because it is piecewise linear in \( \tilde{P}_1 \) (Lapan, Moschini, and Hanson, 1991). In particular, because of the truncated option return, the distribution of \( \pi_b \) will exhibit skewness which violates condition (C1). Therefore, condition (C2) must be tested in order to determine the appropriateness of using the two-parameter model. To test condition (C2), twenty trials of a portfolio simulation were
conducted to derive sample values of the ratio. The GAUSS computer code for this simulation (PORTFOL2.GSS) is contained in the appendix. The sample mean and sample standard deviation over the twenty trials is 0.0121 and 0.0126 respectively. The sample $t$-statistic of 0.96 falls short of the critical value of 1.73 for probability of type I error equal to 0.10 and 20 degrees of freedom. Therefore, the null hypothesis that the true ratio mean is equal to zero is not rejected and condition (C2) is satisfied. This result is consistent with the findings of Hanson and Ladd.5

The certainty equivalent for portfolio $y_b$ is therefore

$$CE_b = E[\pi_b | \Omega] - \frac{1}{2} A \cdot \text{Var}[\pi_b | \Omega].$$

(12)

Because of the nonlinearity $\pi_b$ in $\tilde{P}$, the derivations of $E[\pi_b | \Omega]$ and $\text{Var}[\pi_b | \Omega]$ are quite complex. Using conditional expectations and variances, these moments can be derived as follows6:

$$E[\pi_b | \Omega] = (\mu_p - P_0)I_b - \frac{1}{2} \cdot \gamma \cdot I_b^2 + (\mu_p - P_f)X_b + (P_R - \alpha B_2)R_b,$$

(13)

$$\text{Var}[\pi_b | \Omega] = \alpha \cdot \sigma_f^2 \cdot (I_b + X_b + R_b)^2 + \alpha (1 - \alpha) \cdot \sigma_f^2 \cdot (I_b + X_b)^2$$

$$+ \alpha (1 - \alpha) \cdot [B_1(I_b + X_b) + B_2 R_b]^2,$$

(14)

where

$$\alpha = \text{Prob}[P_1 \leq K | \Omega],$$

$$B_1 = E[\tilde{P}_1 | P_1 > K, \Omega] - E[\tilde{P}_1 | P_1 \leq K, \Omega],$$

$$B_2 = K - E[\tilde{P}_1 | P_1 \leq K, \Omega],$$

$$\sigma_f^2 = \text{Var}[\tilde{P}_1 | P_1 \leq K, \Omega],$$

$$\sigma_f^2 = \text{Var}[\tilde{P}_1 | P_1 \leq K, \Omega].$$

For determining the optimal portfolio, equations (13) and (14) are substituted into equation (12), which yields the following first-order conditions for expected utility maximization:

$$\begin{bmatrix}
\partial EU(\pi_b)/\partial I_b \\
\partial EU(\pi_b)/\partial X_b \\
\partial EU(\pi_b)/\partial R_b
\end{bmatrix} = \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} = \begin{bmatrix}
\mu_p - P_0 - \gamma \cdot I_b^* \\
\mu_p - P_f \\
P_R - \alpha B_2
\end{bmatrix} - A \begin{bmatrix}
V_{11} & V_{11} & V_{12} \\
V_{11} & V_{11} & V_{12} \\
V_{12} & V_{12} & V_{22}
\end{bmatrix} \begin{bmatrix}
I_b \\
X_b \\
R_b
\end{bmatrix},$$

(15)

where

$$V_{11} = \sigma_f^2 = \alpha \cdot \sigma_f^2 + (1 - \alpha) \cdot \sigma_f^2 + \alpha (1 - \alpha) \cdot B_1^2,$$

$$V_{12} = \alpha \cdot \sigma_f^2 + \alpha (1 - \alpha) \cdot B_1 B_2,$$

$$V_{22} = (1 - \alpha) \cdot \sigma_f^2 + \alpha (1 - \alpha) \cdot B_2^2.$$
The third vector in equation (15) is the vector of mean returns to the portfolio. Note that $\alpha B_2$ can be interpreted as the investor's subjective valuation of the put option. It is equal to the probability of the option being exercised, $\alpha$, multiplied by the subjective expected value of the option if it is exercised, $B_2$. The matrix of $V$ terms is the variance-covariance matrix of returns for the portfolio. Note that the returns to inventory and futures have variance and covariance equal to $V_{11}$. This is because inventory and futures have the same settlement price $P_1$ when basis risk is ignored. Also, note that the mean, variance, and covariances of the put option position are functions of the spot price mean and variance. This is because they are determined by the conditional moments of a normal distribution.

The first-order conditions from equations (15) yield the following portfolio equations for inventory, futures, and the put option:

$$r_b = \frac{P_f - P_0}{g},$$

$$x_b^* = \frac{\mu_p - P_f}{AV_{11}} - \frac{V_{12}}{V_{11}} R_b^* - I_b,$$

$$R_b^* = \frac{P_R - \alpha B_2}{AV_{22}} - \frac{V_{12}}{V_{22}} (I_b^* + X_b^*).$$

Equation (16) demonstrates that the inventory demand function remains unchanged from equation (9) when the put option is added to the portfolio. However, the futures demand equation (17) is different from equation (10) because of the addition of a put option hedging component $-\left(\frac{V_{12}}{V_{11}}\right) R_b^*$. The ratio $V_{12}/V_{11}$ represents the number of short futures contracts needed to fully hedge one written put option contract.

Equation (18) demonstrates that the structure of the option demand equation is similar to the structure of the futures demand equation. The speculative component is equal to $(P_R - \alpha B_2)/AV_{22}$ and the hedging component is equal to $(V_{12}/V_{22})(I_b^* + X_b^*)$. The ratio $V_{12}/V_{22}$ is the number of purchased put option contracts needed to fully cover one outright long futures contract.

To derive access value for the put option, the solution to equation (4) gives

$$AV = \frac{(V_1d_R - V_{12}d_K)^2}{2AV_{11}(V_{11}V_{22} - V_{12}^2)},$$

where

$$d_R = P_R - \alpha B_2,$$

$$d_K = \mu_p - P_f.$$

The denominator of equation (19) is always positive; therefore, access value can never be less than zero. Access value is zero only when the following condition is met:
In other words, when the expected marginal speculative return on the option is equal to the hedge adjusted expected marginal speculative return on the futures, then access value is zero for the put option. When these terms diverge, access value increases exponentially.

It is difficult to analytically determine the partial derivatives of equation (19) with respect to the moments of spot price distribution. This is because \( d_R, V_{12}, \) and \( V_{22} \) are complicated functions of both moments. However, it is possible to derive the partial derivative of equation (19) with respect to the investor's risk preferences (i.e., \( A \)). This derivative is

\[
\frac{\partial (AV)}{\partial A} = \frac{-(V_{11}d_E - V_{12}d_\mu)^2}{2A^2V_{11}(V_{11}V_{22} - V_{12}^2)},
\]

which is less than zero. Therefore, increases in absolute risk aversion result in decreases in access value for the put option. This is not surprising, because expected utility models containing options and futures have demonstrated that the futures contract is the preferred instrument for hedging unavoidable risks when transaction costs and quantity risks are ignored (Wolf, 1987; Lapan, Moschini, and Hanson, 1991).

IV. Simulation Analysis

As mentioned in the previous section, the relative influence of spot price moment information in determining the access value of the put option is not easily discerned from equation (19). This is because \( d_R, V_{12}, \) and \( V_{22} \) are functions of both moments. To solve equation (19) for the spot price moments would result in an elaborate expression that would not likely yield useful information. As a substitute for the analytic analysis, a stochastic simulation model of equation (19) was constructed (see PORTFOLI.GSS in appendix) in GAUSS. This model provides a more efficient analysis of the influence of moment information upon option access value.

The content of the investor's information set on the moments was measured using the following distance metrics:

\[
M(\mu_p) = |\mu_p - P_f|,
\]

\[
M(\sigma_p^2) = |\sigma_p^2 - IV^2|,
\]

where \( IV \) is the market implied volatility derived using \( \alpha B_2 \) as the valuation formula. Equations (20) and (21) imply that the content of the investor's information set is measured by the distance of the private moment estimate from the market estimate.

February 20, 1991 wheat and corn prices were used in the simulation model for the period zero prices (i.e., \( P_0, P_f, \) and \( P_R \)). Table 1 lists these prices along with the corresponding strike prices and market implied volatilities. For futures prices, the July 1991 KCBOT was used for wheat and the December 1991 CBOT was used for corn. The spot prices were taken from
Table 1. Price and Volatility Data for Simulation Model

<table>
<thead>
<tr>
<th>Code</th>
<th>Commodity</th>
<th>Futures</th>
<th>Spot</th>
<th>Strike</th>
<th>Option</th>
<th>Implied Volatility</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>WHEAT</td>
<td>2.7275</td>
<td>2.73</td>
<td>2.60</td>
<td>0.0525</td>
<td>0.2753</td>
</tr>
<tr>
<td>1B</td>
<td>WHEAT</td>
<td>2.7275</td>
<td>2.73</td>
<td>2.70</td>
<td>0.0975</td>
<td>0.2565</td>
</tr>
<tr>
<td>1C</td>
<td>WHEAT</td>
<td>2.7275</td>
<td>2.73</td>
<td>2.80</td>
<td>0.1500</td>
<td>0.2586</td>
</tr>
<tr>
<td>2A</td>
<td>CORN</td>
<td>2.5825</td>
<td>2.48</td>
<td>2.50</td>
<td>0.1350</td>
<td>0.4086</td>
</tr>
<tr>
<td>2B</td>
<td>CORN</td>
<td>2.5825</td>
<td>2.48</td>
<td>2.60</td>
<td>0.1850</td>
<td>0.4538</td>
</tr>
<tr>
<td>2C</td>
<td>CORN</td>
<td>2.5825</td>
<td>2.48</td>
<td>2.70</td>
<td>0.2400</td>
<td>0.4571</td>
</tr>
</tbody>
</table>

The par delivery markets (Kansas City and Chicago) for the futures contracts. The option strike prices were chosen to get one each in the in-, at-, and out-of-the-money range.

Values for the moments of the investor's information set \( \Omega \) were randomly generated from an underlying probability distribution that was assumed to be normal with mean equal to the market estimate of the moment and a standard deviation equal to ten percent of the mean. The investor's risk aversion coefficient \( \alpha \) was randomly generated from a uniform distribution with values ranging from 0.000001 for the minimum to 0.01 for the maximum.

For each set of price data, 500 simulation trials were run. To obtain a measure of the magnitude of option access value, a geometric mean over trials was calculated for the following measure:

\[
M(AV) = \frac{AV}{CE_a} 
\]

Note that \( M(AV) \) can be interpreted as the percentage increase in value to the investor when the put option is introduced into the portfolio. To measure the relative influence of preferences and information metrics upon access value, the following regression equation was estimated for each set of price data:

\[
\ln(AV) = \beta_0 + \beta_1 \ln(\alpha) + \beta_2 \ln(M(\mu)) + \beta_3 \ln(M(\sigma^2)) + u, \tag{22}
\]

where \( u \) is a normally distributed random variable with mean equal to zero and variance equal to \( \sigma^2 \). The logarithmic form of equation (22) implies that the regression coefficients represent elasticities for the independent variables.

Table 2 summarizes the simulation results for each set of price data. The geometric means of the percentage increase range from 6.1 to 106.8 percent. No relationship is apparent between these and the strike prices of the options. Both risk aversion and the mean information metric are highly significant in all equations. A one percent increase in risk aversion causes approximately a one percent decrease in access value. A one percent increase in the mean information metric causes increases in access value that range from 1.43 to 2.28 percent.
### Table 2. Summary of Simulation Results
(Standard Errors in Parentheses)

<table>
<thead>
<tr>
<th>Variable</th>
<th>1A</th>
<th>1B</th>
<th>1C</th>
<th>2A</th>
<th>2B</th>
<th>2C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric Mean</td>
<td>20.4%</td>
<td>106.8%</td>
<td>91.2%</td>
<td>28.2%</td>
<td>6.4%</td>
<td>6.1%</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>1.294</td>
<td>1.239*</td>
<td>2.183**</td>
<td>-1.159*</td>
<td>0.163</td>
<td>-1.828**</td>
</tr>
<tr>
<td>(0.851)</td>
<td>(0.696)</td>
<td>(0.700)</td>
<td>(0.603)</td>
<td>(0.760)</td>
<td>(0.888)</td>
<td></td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>-0.914**</td>
<td>-1.005**</td>
<td>-1.106**</td>
<td>-0.981**</td>
<td>-0.931**</td>
<td>-1.012**</td>
</tr>
<tr>
<td>(0.112)</td>
<td>(0.089)</td>
<td>(0.088)</td>
<td>(0.083)</td>
<td>(0.104)</td>
<td>(0.122)</td>
<td></td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>2.284**</td>
<td>1.851**</td>
<td>2.197**</td>
<td>1.608**</td>
<td>1.530**</td>
<td>1.428**</td>
</tr>
<tr>
<td>(0.107)</td>
<td>(0.066)</td>
<td>(0.072)</td>
<td>(0.071)</td>
<td>(0.105)</td>
<td>(0.110)</td>
<td></td>
</tr>
<tr>
<td>( \beta_3 )</td>
<td>-0.004</td>
<td>-0.100</td>
<td>0.085</td>
<td>-0.141**</td>
<td>0.489**</td>
<td>0.202*</td>
</tr>
<tr>
<td>(0.106)</td>
<td>(0.077)</td>
<td>(0.085)</td>
<td>(0.069)</td>
<td>(0.095)</td>
<td>(0.116)</td>
<td></td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.52</td>
<td>0.65</td>
<td>0.68</td>
<td>0.57</td>
<td>0.40</td>
<td>0.34</td>
</tr>
<tr>
<td>( \Gamma(4,496) )</td>
<td>176.6**</td>
<td>306.7**</td>
<td>358.4**</td>
<td>219.3**</td>
<td>110.4**</td>
<td>85.9**</td>
</tr>
<tr>
<td>DW</td>
<td>2.10</td>
<td>1.90</td>
<td>2.11</td>
<td>2.03</td>
<td>2.04</td>
<td>1.96</td>
</tr>
</tbody>
</table>

**Notes:**  
*Significant at 90 percent confidence level.  
**Significant at 95 percent confidence level.

The influence of the variance information metric upon access value appears to be mixed and only highly significant in two equations. In some cases, the increase in the metric results in an increase in access value. In other cases, the increase in the metric results in a decrease in access value. Equation (19) indicates that variance can have two opposing effects upon access value: a return effect in the numerator and a risk effect in the denominator. As mentioned previously, increases in risk will cause the investor to favor futures over options. Therefore, the sign of the coefficients for the variance metric will be positive if the return effect dominates the risk effect, and will be negative if the risk effect dominates the return effect. Note that this implies that there may exist an optimal level of expected price variance at which the access value of the option is maximized.

The previous results provide a direct extension, to option markets, of the futures market research of Grossman (1977) and Brannen and Ulveling (1984). Grossman’s theoretical model provided the result that futures markets would only exist if the current market prices are a sufficiently inaccurate forecast of future spot prices. In particular, a “noisy” rational expectations equilibrium is a necessary condition for the existence of a futures market. Brannen and Ulveling empirically tested Grossman’s theoretical result by regressing past prices (public information) on future prices for several commodities. One of their conclusions was that significant information differentials were needed to support the existence of futures trading on commodity markets.
The previous results of this section show that an investor's private value of an option market is closely correlated with the absolute differential between their private information on the moments and the public estimates. If private value is positively correlated with market viability, then option markets will only exist for those commodities which have sufficient "noise" in the pricing system to induce significant heterogeneous expectations among investors.8

V. Summary

In this paper, access value is defined as the amount an investor is willing to pay in order to have access to a derivative market. This value is conditional upon the content of the investor's information set. Futures options are a derivative asset that allow commodity investors to more efficiently utilize their information on the moments of the future spot price distribution.

In Section II, a money-metric measure of access value is constructed using certainty equivalents. This measure is consistent with expected utility theory if the investor's utility function satisfies the von Neumann-Morgenstern axioms. A procedure is derived to test the sufficiency of using two-parameter certainty equivalent models in measuring access value.

In Section III, theoretical portfolio models are constructed for a commodity investor with constant absolute risk aversion. The first portfolio contains spot and futures markets. The second portfolio model contains spot, futures, and a put option market. The investor's information set is assumed to contain estimates of the first two central moments of the underlying spot price distribution.

The theoretical access value of the put option is derived by taking the difference between certainty equivalents for the two portfolios. Access value is zero if the hedge adjusted marginal expected speculative returns to the futures are equal to the marginal expected speculative returns to the option. Divergence in these terms results in exponential increases in access value. Another result from the analytical model is that increases in risk aversion result in decreases in access value.

In order to examine the relative influence of the investor's information set upon access value, simulation trials of the portfolio model were conducted in Section IV. The investor's moment information set content was measured using distance metrics. These metrics measured the distance of the investor's private estimate (generated randomly from a normal distribution) from the market estimate (using actual prices) of the moment. An increase in the metric for information on the mean increased the access value of the option by an approximate factor of two. The results for the variance metric were mixed. One possible hypothesis for this observation is there exists a tradeoff between a return and a risk effect. When the return effect is dominant, increases in the metric increase access value. When the risk effect is dominant, increases in the metric decrease access value. Note that this implies that there may exist an optimal level of variance that maximizes access value given an estimate of the spot price mean.
GAUSS Computer Code for Simulation Models

PORTFOL2.GSS

fp = unfrngen(1.50,2.00,1);
k = unfrngen(1.50,2.00,1);
pr = (k>fp)*(k-fp);
pl = unfrngen(1.50,2.00,1);
ep2 = unfrngen(1.50,2.00,1);
v11 = unfrngen(0.05,.50,1);
risk = unfrngen(.000001,.01,1);
g = .005;

z = (k-ep2)/(v11^0.5);
alpha1 = cdfn(z);
alpha2 = cdfnc(z);
mills1 = pdfn(z)/alpha1;
mills2 = pdfn(z)/alpha2;

v23 = alpha1*alpha2*b2*b3;
v22 = alpha1*cvm+alpha1*alpha2*b2^2;
v33 = alpha2*cvg+alpha2*alpha2*b3^2;
v12 = v22+v23;
v13 = v33+v23;
sigma1 = (ep2-fp)/(risk*v12);
sigma2 = (hr-alphal*b2)/(risk*v22);

beta1 = v11/v12;
beta2 = v12/v22;
i = (fp-p1)/g;

r = (sigma2*beta1-sigma1*beta2)/(beta1-beta2);
x = (ep2-fp)/(risk*v11)-(v12h11)*r-i;
p2 = (1.50 1.55 1.60 1.65 1.70 1.75 1.80 1.85 1.90 1.95 2.00);
p2 = p2^2';

y2 = (p2-p1)^i-0.5*g*i^2+(p2-fp)*x+(tr-(k.p2).*(k-p2))^r;
muy = mean(y2);
cubey = (y2-muy)^3;
skewy = mean(cubey);

percnt = (((risk^2/6)*skewy/muy)*100;
print "Percent Skewness =";
print percnt;
end:
PORTFOLIOSS

print "Input Futures Price";
fp = con(1,1);
print "Input Implied Volatility";
iv = con(1,1);
print "Input Option Premium";
pr = con(1,1);
print "Input Option Strike Price";
k = con(1,1);
print "Input Local Cash Price";
p1 = con(1,1);
print "Number of Iterations?";
ni = con(1,1);
a = unfrmgen(.000001,.01,ni);
g = .005;
ep2 = normgen(fp,1*fp,ni); 10
v11 = normgen(iv^2,1*iv^2,ni);
z = (k-ep2)/(v11^0.5);
alp1 = cdfn(z);
alp2 = cdfnc(z);
mills1 = pdfn(z)/alp1;
mills2 = pdfn(z)/alp2;
cm1 = ep2-mills1.*(v11^0.5);
cmg = ep2+mills2.*(v11^0.5);
cv1 = (1-z.*mills1-mills1^2).*v11; 11
cvg = (1+z.*mills2-mills2^2).*v11; 11
b1 = cmg - cm1;
b2 = k - cm1;
b3 = cmg - k;
v23 = alp1.*alp2.*b2.*b3;
v22 = alp1.*cv1+alp1.*alp2.*b2^2;
v33 = alp2.*cvg+alp1.*alp2.*b3^2;
v12 = v22+v23;
v13 = v33+v23;
sigma1 = (ep2-fp)./(a.*v12);
sigma2 = (pr-alp1.*b2)./(a.*v22);
beta1 = v11./v12;
beta2 = v12./v22;
i1 = (fp-p1)/g;
x1 = (ep2-fp)./(a.*v11)-i1;
mu1 = (ep2-p1).*i1-0.5*2^2+(ep2-fp).*x1;
var1 = v11.*(i1+x1)^2;
ycel = mu1-0.5*a.*var1;
i2 = i1;
r2 = (sigma2.*beta1-sigma1.*beta2)/(beta1-beta2);
x2 = (ep2-fp)./(a.*v11)-(v12./v11).*r2-i2;
mu2 = (ep2-p1).*i2-0.5*2^2+(ep2-fp).*x2+(pr-alp1.*b2).*r2;
\[ \text{var2} = \alpha_1 \cdot c_1 \cdot \left( (i^2 + x^2 + r^2) \right)^2 + \alpha_2 \cdot c_2 \cdot \left( (i^2 + x^2) \right)^2 + \alpha_1 \cdot \alpha_2 \cdot (b_1 \cdot (i^2 + x^2) + b_2 \cdot r^2) \]

\[ \text{yce2} = \mu_2 - 0.5 \cdot a \cdot \text{var2} \]

\[ \text{value1} = \ln \left( \frac{\text{yce2} - \text{ycel}}{\text{ycel}} \right) \]

\[ \text{vmean} = \exp \left( \text{mean} \left( \text{value1} \right) \right) \]

print "Mean Percentage Increase";
print vmean*100;

\[ \text{value2} = \ln \left( \frac{\text{yce2} - \text{ycel}}{\text{ycel}} \right) \]

\[ \text{indxl} = \ln \left( \text{abs} \left( \text{ep2-fp} \right) \right) \]

\[ \text{indx2} = \ln \left( \text{abs} \left( \text{vi1-iv} \left( ^2 \right) \right) \right) \]

\[ a = \ln \left( a \right) \]

\[ \text{data1} = \text{value2} \]

\[ \text{data2} = a' - \text{indxl}' - \text{indx2}' \]

\[ _\text{altnam} = \{ "\text{CONST}", "A", "\text{MEAN}", "\text{VAR}", "\text{VALUE}" \} \]

\[ _\text{olsres} = 1 \]

call ols(0, data1, data2);

\[ \text{end} \]

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The authors acknowledge the helpful comments of Jay Coggins, Mike Kehoe, Myles Watts, and two anonymous reviewers.

**Notes**

1. This is dependent upon the assumption that the investor is not constrained with regard to participation in the added opportunities.

2. In this case, the derivative security is considered to be unique.

3. See, for example, Robison and Barry (1987) for a complete description of the two-parameter certainty equivalent model and its usage for various problems in economics and finance.

4. In theory, the market estimate of the futures price \( P_f \) can be downwardly (upwardly) biased due to a net short (long) hedge open interest in the market (Stein, 1986; Hirshleifer, 1988). An investor may incorporate this known bias into his/her estimate of the future spot price mean \( \mu_p \) which would result in differences between \( P_f \) and \( \mu_p \). This is conditional upon the investor having information on the net hedging open interest and knowledge of the functional form of the bias.

5. The debate over the appropriateness of using mean-variance (EV) models instead of expected utility maximization is evident in the literature for more than 20 years with prestigious authors on both sides (Borch, 1969; Feldstein, 1969; Tobin, 1969; Levy and Markowitz, 1979). This debate has recently been extended to the usage of EV models for portfolios containing commodity options. Lapan, Moschini, and Hanson (1991) questioned the appropriateness of the EV framework with options by showing an inconsistency between positions implied by EV analysis and expected utility maximization. However, Bullock and Hayes (1992) used an endogenous representation of the variance-covariance matrix of portfolio returns to show that EV analysis can give optimal positions that are consistent with expected utility maximization. In addition, Hanson and Ladd (1991) used simulation analysis to compare EV analysis with expected utility maximization for the case of truncated distributions. They found that the EV model gave results that were robust to those of expected utility maximization.

7. The functional form of the model was determined by estimating a full Box-Cox transformation of the regression equation using data generated from sample 1A in Table 1. The estimation produced a maximum likelihood transform parameter estimate of 0.04 which was rounded to zero. This value of the parameter corresponds to the logarithmic functional form.

8. Perhaps this explains why we observe a high correlation between successful futures contracts and subsequently successful introduction of option contracts in commodity markets. If information differentials are not sufficient to support a futures contract, then they are not sufficient to support an option contract.

9. Unfrmgenn(a,b,n) is a user-defined function that provides a n x 1 matrix of random numbers with a uniform distribution with minimum equal to a and maximum equal to b.

10. Normgen(m,s,n) is a user-defined function that generates a normal random variable with mean equal to m and standard deviation equal to s.

References


