Competition Between Open Source and Closed Source Software: The Contribution of Users

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Abstract

I consider a model of vertical product differentiation with consumers contributing to the quality of an open source software and study the impact of open source in a market where a single firm sells a closed source software. The open and closed source software products may coexist in the market, but if contributions to the open source software are significant, the closed source vendor may lower its price to a level that forces the open source out of the market. The firm’s lower price not only increases demand for its software, but also induces consumers into switching from open to closed source software therefore reducing the contribution of users. From a welfare point of view, open source entry always increases consumers’ surplus; however, closed source entry on a market with open source software may decrease consumers’ surplus.

Keywords: Location Model; Network effects; Open source; Competition; Software industry structure

JEL classification: D23; D43; H44; L11; L13; L17; L19; L86; O38

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1 Introduction

Two paradigms coexist in the software industry: the open source and the closed source. The former is characterized by a shared code to which anyone can contribute, while the latter has a proprietary code to which only the owner has access. Open source software may be produced through decentralized collaboration. For example, university professors created the statistical software R via contributions from outside the corporate framework.\(^1\) The nature of open source software allows users to contribute their expertise and knowledge directly to the software. Furthermore, these contributions circulate rapidly among users at practically no cost [Demazière et al., 2007].

This paper focuses on user contributions and its impact on the competing environment between closed and open source software. More specifically, I consider three topics: (i) the effect of user contributions to open source software on the industry’s market structure, (ii) the impact on welfare from the entry of either closed or open source software into the market, (iii) the consequences of price discrimination on the aforementioned effects.

Early research on this topic has focused on what motivates individuals to freely contribute to open source software; see for example Lakhani and Wolf [2005], Harhoff et al. [2003], and Lerner and Tirole [2005a], or for a review of the literature see von Krogh and von Hippel [2006]. Previous discussions such as Lerner and Tirole [2005b], Economides and Katsamakas [2006], and Scotchmer [2010] explored specific characteristics of open source relative to closed source software. However, direct competition between open and closed source software has received little consideration. Sen [2007], Jullien and Zimmermann [2008], Lin [2008], Lanzi [2009], and Casadesus-Masanell and Ghemawat [2006] study the occurrence, in a closed source software dominant market, of an open source software entrant.

While closed source products are, in general, well advertised and documented, such benefits seldom pertain to open source products. In fact, most consumers may not be

\(^1\)Consumers increase a software value by increasing its reliability. Their contribution is twofold: the code contains fewer errors and has increased execution speed. Contributors provide the installation, integration, and maintenance services necessary to operate the open source software efficiently and reliably.
aware of the existence of an open source solution to their needs. Taking this into consideration, Sen [2007] assumes that acquiring information regarding open source software is part of its cost. Closed source users, however, have no such cost. He equally suppose that both open and closed source software products benefit from a direct network effect. His argument considers the competition between a seller of closed source software and a vendor of services for open source products. He demonstrates that the vendor of services benefits from the open source having a higher cost or lower usability — the ease of use and the conveniences of software — than the closed source software.

Jullien and Zimmermann [2008] also study competition, but allow users to influence the open source software’s quality. On one side, the closed source vendor chooses between two levels of quality having different fixed costs. On the other side, an open source provider, given that it invests a minimal amount, benefits from improvement done by users. These contributions determine the quality of the open source software. The authors derive the conditions under which a firm decides to invest in an open source project. Low skilled users motivate the firm to invest little, thus resulting in a lower quality product targeting a relatively price-sensitive market. In the presence of mainly highly skilled users the firm chooses a large and increasing investment. Consequently, it benefits from a high quality product and generates a profit that increases with the skill set of its consumers.

In another duopoly model, Lin [2008] considers a closed source firm competing with an open source distributor, in which consumers differ with respect to skill and experience. She demonstrates that open source may come to dominate the market when its consumers derive significant benefits from this software. However, when the open source software does not provide sufficient benefits to skilled consumers, the mere fact that it is free does not guarantee its success nor its survival.

Using vertical differentiation to study competition, Lanzi [2009] models consumers that face positive switching costs and differ in their ability to use software. Like Meng

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2Since the source code of open source software is available for free, most firms distributing open source software offer technical support and services to consumers. These services are an important revenue source for such a firm.
and Lee [2005], he considers a community of users interested in maximizing, not profit, but the number of open source users. The model is a two-stage game. In the first stage two entities select the quality level of their software, and in the second stage a closed source software licensor sets his price. He concludes that skilled users always choose the open source software. If switching costs are low, then upon entry, the closed source firm lowers its price below the monopoly level. If switching costs are high, the closed source licensor increases its price relative to its monopoly price, thus compensating for its decreased market share. In fact, Lanzi [2009] finds that a portion of the market is always captured by a high value open source software entrant.

Casadesus-Masanell and Ghemawat [2006] examine competition between a firm owning a closed source software and some entity distributing an open source software. They focus on competition between Linux (an open source software) and Windows (a closed source software). Two assumptions of their model is that some users are constantly entering and exiting the market and that both open and closed source software benefit from a direct network effect. However, the repercussions differ for the two types of firms. Specifically, they find that the closed source software’s price (Windows’ price) is greater under monopoly than under competition from the open source (Linux). They also claim that the two software types may coexist in equilibrium. Their study suggests that a closed source software monopoly may yield higher welfare than a duopoly. The monopoly outcome may dominate the duopoly because the entry of Linux induces some consumers to switch from Windows to Linux. Fewer Windows’ consumers implies lower benefits from network effect for that firm. If this network effect is sufficiently large, the closed source monopoly outcome may, in terms of welfare, outweigh the duopoly outcome.

The framework of this paper considers a closed source software firm exposed to competition from an open source product. While the firm depends on its intrinsic quality to create value, the open source software derives value from user contributions. The open source software may present a lower or higher quality than the closed source software. This difference in quality generates two distinct equilibria, which I treat independently.
First, I suppose that the closed source firm knows only the distribution of tastes but cannot distinguish among buyers; then I relax this assumption by assuming that the firm can perfectly identify each consumer’s taste. The latter allows the firm to engage in first-degree price discrimination.

The outcome of the model shows that as contributions to open source software increase, consumers may turn to the closed source rather than to the open source product. This is true whether or not the firm price discriminates. The closed source software may even entirely dominate the market because the competition induces the firm to lower its price. In fact, the closed source software licensor subsidizes its own product in order to lower demand for the open source product. Lowering its price not only makes its product more attractive, it reduces consumer contributions to the open source. Consumers are not only impelled to purchase the closed source software they also do not contribute to the open source product.

When the firm engages in price discrimination, one may observe an equilibrium in which it sells to some consumers at a negative price; in effect, the firm is paying certain consumers to use its software. This is in fact an extreme case of the result just discussed: price discrimination magnifies the previous effect.

I also concluded that the entry of a closed source software competing with an open source product may lower welfare, even if the firm price discriminates. However, the entry of an open source software always increases welfare.

The paper is organized as follows. First, as a benchmark, I study the monopoly case for a closed and for an open source software. Second, I analyze how a closed source software firm competes with an open source software when the quality of the former is higher than the quality of the latter. Then, I determine how the equilibrium is affected when the firm price discriminates. Subsequently, I look at the outcome of competition when the ranking of quality is reversed. The final section summarizes the major findings.

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4If the firm practices uniform pricing it lowers the price for all consumers; but if the firm discriminates, it lowers the price for the consumers with a high valuation of quality.
2 Monopoly

The first baseline has a single firm offering a closed source software of quality \( v_c \). In the alternative baseline, open source software of quality \( v_o \) is the only product available in the market. Both types of software are produced at zero cost. The utility of consumer indexed \( \sigma \) is \( U(\sigma) = \sigma v_i \) where \( i = c, o \), and the taste parameter \( \sigma \) is uniformly distributed on \([0, 1]\). All consumers buy at most one unit. Not consuming the software yields a utility of zero.

2.1 A closed source monopoly

I consider two equilibria: one with uniform pricing, the other with price discrimination.

2.1.1 Uniform pricing

When the firm’s software is sold at price \( P \), it is purchased by all consumers whose surplus \( S_c(\sigma) = \sigma v_c - P \) is non-negative. The marginal consumer has index \( \sigma = \sigma_c \) where

\[
\sigma_c(P) = \frac{P}{v_c}. \tag{1}
\]

Demand for the closed source software is \( 1 - \sigma_c(P) \). Since cost is zero, the profit is \( \Pi_m = P(1 - \sigma_c(P)) \). The profit maximizing price \( P^*_m = v_c / 2 \) yields an equilibrium market size of \( X^*_c = 1/2 \).

2.1.2 Discriminatory pricing

When the monopolist sells at personalized price \( P(\sigma) \), the consumer indexed \( \sigma \) has surplus \( S_c(\sigma) = \sigma v_c - P(\sigma) \). All consumers with a non-negative surplus purchase the software. In fact, the firm sets price \( P(\sigma) = \sigma v_c \) to consumer of type \( \sigma \) and, thus, captures the entire surplus.

\[^{5}\text{This result is equivalent to the result of Mussa and Rosen [1978] in their monopoly example.}\]
2.2 An open source monopoly

The open source software differs from the closed source software in that its value to any particular user depends on the total number of users. Consumers consider the expected size of the open source software’s market when they decide whether or not to use this software. I do not model how consumers’ expectations are formed. However, I impose the restriction that consumers’ expectations are fulfilled in equilibrium. Each consumer correctly anticipates the total number of consumers that will use the open source software.\footnote{In this regard, I follow Katz and Shapiro [1985].} I denote the expected size of the market for open source software by $X^e_o$.

I let $\psi$ denote the cost of learning how to use the open source software. The open source software is, then, used by all consumers whose surplus $S_o(\sigma, X^e_o) = v_o \sigma - \psi + kX^e_o$ is non-negative,\footnote{The linear relationship between consumers’ expectations of the network size and the surplus is in accordance with research in network economics, for example, Katz and Shapiro [1985] and Shy [2001].} where the parameter $k > 0$ represents the marginal contribution of each user to the benefit that others derive from the software. Following Katz and Shapiro [1985], I refer to $\phi = \psi - kX^e_o$ as the hedonic cost of open source software. This is the cost adjusted for the effect of user contributions. I will henceforth refer to $k$ as contributions.

The following assumption is made throughout the paper:

$$v_o > \psi > k \quad (2)$$

This assumption (2) implies the following: a) The consumer indexed $\sigma = 0$, has a negative surplus from the open source software, even when he expects all consumers to use it; b) the consumer indexed $\sigma = 1$, has a positive surplus from the open source software, even when he expects no consumer to use it; and c) all consumers face a strictly positive hedonic cost, or more exactly, $\psi - kX^e_o = \phi > 0$.

The consumer indifferent between getting the open source software and choosing
the outside option has a taste parameter

$$\sigma_o = \frac{\psi - k X_o^e}{v_o} = \frac{\phi}{v_o}.$$  \hspace{1cm} (3)

Thus, the demand for open source software is $$X_o(X_o^e) = 1 - \sigma_o(X_o^e).$$

Figure 1 illustrates the fulfilled expectations condition

$$X_o = X_o^e.$$ \hspace{1cm} (4)

The abscissa represents consumers’ expectations, and the ordinate represents their corresponding demand. The line shown as $$X_o(X_o^e)$$ is the actual demand for the open source product when consumers expect $$X_o^e$$. Assumption (2) ensures

$$\frac{\partial X_o}{\partial X_o^e} = \frac{\partial (1 - \sigma_o(X_o^e))}{\partial X_o^e} = \frac{k}{v_o} \leq 1.$$ \hspace{1cm} 8

Using $$X_o^e = X_o(X_o^e)$$ and (3) yields the equilibrium market size:

$$X_o = \frac{v_o - \psi}{v_o - k} \in (0, 1).$$  \hspace{1cm} (5)

Note the difference with the closed source software equilibrium. The market size is now a function of parameters $$v_o, \psi,$$ and $$k$$. Note in particular that the market size is increasing in $$k$$. Whereas the size of the market of an open source monopoly increases in $$v_o$$, the size of the market of a closed source monopoly is always equal to $$1/2$$. The

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8 A stability argument can be invoked to justify that the actual demand has slope less than one. To see why, consider the following example. If a consumer underestimates demand (the demand he expects is located to the left of the point depicted as $$X_o^e = X_o$$ along the horizontal axis), he observes that the actual demand lies on the hard line above his expectation represented by the $$45^\circ$$ line. Realizing that he underestimates demand, the consumer updates his expectation. His new expectation is the point on the $$45^\circ$$ line that corresponds to the actual demand, that is, his updated expectation still lies below the actual demand but is closer than the expectation he made before the update. After doing this an infinity of time, the consumer converges to the fixed point where his expected demand equals actual demand. If the actual demand function is steeper than the $$45^\circ$$ line, the consumer updating his expectation would reach either expected demand of zero or expected demand of infinity.
size of the market for open source is larger (smaller) than $1/2$ when $\upsilon_o + k > (\leq) 2\psi$.\(^9\)

In terms of market size, when $\upsilon_o$ is low, closed source software performs better than open source software; and when $\upsilon_o$ is high and contributions are significant, open source software performs better than closed source software. In the next two sections, I study a situation in which the firm competes against an open source software.

\(^9\)Remark that assumption (2) is sufficient to ensure that the equilibrium market size lies in the set $(0, 1)$.\]
3 Duopoly: High-quality closed source software

I now assume that closed and open source software compete in the market place and that $v_c > v_o$. I also assume that consumers form their expectations of the number of open source users on the basis of the price or prices set by the closed source firm. I examine in turn the case of uniform\(^{10}\) and discriminatory pricing.

3.1 Uniform pricing

The consumer indifferent between choosing the open source software and buying the closed source software has a taste parameter $\tilde{\sigma}$ that satisfies $S_c(\tilde{\sigma}) = S_o(\tilde{\sigma}, X^e_o)$. Thus,

$$\tilde{\sigma}(X^e_o, P) = \frac{P - \phi}{v_c - v_o}, \quad (6)$$

where $P$ denotes the uniform price. In an equilibrium in which both products are consumed, consumers are separated into three groups:

- those with indices $\sigma \in [0, \sigma_o)$ do not use any software,
- those with indices $\sigma \in [\sigma_o, \tilde{\sigma})$ choose the open source software, and
- those with indices $\sigma \in [\tilde{\sigma}, 1]$ purchase the closed source software.

Thus, the marginal consumers indexed $\sigma_o$ and $\tilde{\sigma}$, defined by equations (3) and (6) respectively, determine demand for open source software which is given by:

$$X_o(X^e_o, P) = \tilde{\sigma}(X^e_o, P) - \sigma_o(X^e_o). \quad (7)$$

Inserting (3) and (6) into (7) yields

$$X_o(X^e_o, P) = \frac{P}{v_c - v_o} - \frac{\psi}{v_o(v_c - v_o)} + kX^e_o \frac{v_c}{v_o(v_c - v_o)}. \quad (8)$$

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\(^{10}\)If a second closed source software of quality $v_c$ were to enter and act as a Bertrand competitor, it would drive prices down to the marginal cost, zero (from the usual Bertrand argument). Consequently, I do not investigate the matter further.
Differentiating this demand with respect to $X^e_o$ gives $\frac{\partial X^e_o}{\partial X^e_o} = -\frac{k v_c}{v_o(v_c - v_o)}$ which is less than unity if, and only if,

$$v_o(v_c - v_o) - k v_c > 0.$$ 

(9)

This condition ensures the stability of the fulfilled expectations condition for the duopoly.

**Choosing price**

I note first that the firm can choose a price $P^H_a$ which ensures that the demand for open source software is zero. Specifically,

**Lemma 1.** For all $P \leq P^H_a = \psi \frac{v_c}{v_o}$, consumers expect a market of size zero for the open source software, that is $X^e_o = 0$.

**Proof.** The price $P^H_a$ is the $P$ that solves $X_o(0, P^H_a) = 0$. If $P^H_a$ induces a zero market share, so also do all lower prices because $\tilde{\sigma}(X^e_o, P)$ is decreasing in $P$ and $\sigma_o(X^e_o)$ is independent of $P$ as shown by (3) and (6).

Figure 1 illustrates lemma 1. When $P = P^H_a$ the intersection of the 45° line and the actual demand is at the origin. Note that the maximum price which excludes the open source software from the market increases in $\psi$. This is because the cost of learning about open source software lowers the value of open source software.

To determine the price the firm actually sets under duopoly, I note that the demand for the closed source software is $X_c(P) = 1 - \tilde{\sigma}(X^e_o, P)$ for $P > P^H_a$ where $\tilde{\sigma}(X^e_o, P)$ satisfies (6). It then follows from the fulfilled-expectations condition, (4), that

$$X_c(P) = 1 - \frac{P(v_o - k) - v_o \psi}{v_o(v_c - v_o) - k v_c}.$$ 

(10)

Assumption (9) ensures that the denominator is positive. Thus, the demand for closed source software is decreasing in $k$ for $P > P^H_a$, and since lemma 1 implies that the firm never chooses $P < P^H_a$, it is indeed decreasing for any price that the firm would choose. This is because the demand for closed source software depends on the consumers’ expectations of the size of the market for open source software. An increase in $k$ increases the value of the open source software, which in turn lowers demand for the closed source software because more consumers opt for the open source software.
I can now determine the price the firm sets when facing the demand given in (10). As cost is zero, the firm’s profit is, \( \Pi_c^H (P) = PX_c (P) \). The profit is maximum when price\(^{11} \) is given by (11) below\(^{12} \)

\[
p_c^H = \frac{v_c}{2} - \frac{v_o (v_o - \psi)}{2 (v_o - k)}.
\]  

(11)

The market size \( X_c^* \) increases in \( k \) and increases in \( \psi \). This follows from inserting the price given in equation (11) into the demand function given in equation (10) and differentiating the ensuing demand with respect to \( k \) which results in \( \frac{\partial X_c}{\partial k} = \frac{v_c v_o \psi}{2 (v_o (v_c - v_o) - kv_c)^2} > 0 \).

I now turn to the question for what values of the parameters \( P_c^H > P_a^H \).

**Lemma 2.** When \( k < \bar{k}(\psi) \), where \( \bar{k}(\psi) = v_o \left( 1 - \frac{v_o \psi}{v_c (v_o - 2 \psi)} \right) \), the firm sets price \( P_c^H \).

**Proof.** When both software products are in the market, the firm earns a profit:

\[
\Pi_c^H = \frac{(v_o (v_c - v_o + \psi) - kv_c)^2}{4 (v_o - k) (v_o (v_c - v_o) - kv_c)}
\]  

(12)

When \( P = P_c^H \) it earns a profit:

\[
\Pi_a^H = P_a^H \left( 1 - \sigma_c (P_a^H) \right) = \psi \frac{v_c}{v_o} \frac{(v_o - \psi)}{v_o}
\]  

(13)

Comparing equation (12) and equation (13) shows that the profit is larger when \( P = P_c^H \) than when \( P = P_a^H \) if \( k < \bar{k} \).

I can now determine how parameter values, \( k \) in particular, determine the number of open source users. As seen, contributions may induce the firm to set a price that

\(^{11}\)Note that \( P_c^H \big|_{\psi \rightarrow \infty} = P_a^H \), implying that price is a continuous function of \( k \).

\(^{12}\)In an equilibrium where the two software products are in the market, that price is unique since, by assumption (9), \( \frac{\partial^2 \Pi_c^H}{\partial P^2} = -\frac{2 (v_o - \psi)}{v_c (v_o - v_c - kv_c)} < 0 \). The function \( \Pi_c^H \) is twice continuously differentiable with respect to \( P \) (over the interval \( P \in (P_a^H, \infty) \) which implies that \( \Pi_c^H \) is strictly concave if, and only if, \( \frac{\partial^2 \Pi_c^H}{\partial P^2} < 0 \). Since \( P_c^H \) is such that \( \frac{\partial \Pi_c^H}{\partial P} (P_c^H) = 0 \), by definition of concavity, Since \( \Pi \) is differentiable, it is strictly concave if and only if for each \( P_1, P_2 \) on the relevant interval: \( (P_1 - P_2) \frac{\partial \Pi_c^H}{\partial P}(P_1) - \Pi_c^H (P_1) < \Pi_c^H (P_2) \), \( \Pi_c^H (P) > \Pi_a^H (P) \) for all \( P \neq P_c^H \). Furthermore, this price decreases in \( k \) and increases in \( \psi \). The effect of \( k \) on price becomes stronger for larger \( k \) as \( \frac{\partial^2 P}{\partial k^2} = -\frac{v_c (v_o - \psi)}{(v_o - k)^3} < 0 \). On the contrary, the effect of \( \psi \) becomes stronger for larger \( k \) as \( \frac{\partial^2 P}{\partial \psi^2} > 0 \).
reduces the open source market size to zero.

Using the fulfilled-expectations condition in equation (7) and differentiating with respect to $k$ yields

$$
\frac{\partial X_o(P(k))}{\partial k} = \frac{v_c}{(v_o(v_c - v_o) - kv_c)^2} + \frac{v_o}{(v_o(v_c - v_o) - kv_c)} \left( \frac{\partial P(k)}{\partial k} \right)
$$

(14)

This shows that the total effect of $k$ is the sum of two components. The first component is the effect of contributions on consumers’ expectations of demand. I call this the consumers’ expectations effect. The second component is the effect of contributions on price. I refer to the latter as the price effect. The first component is positive and the second component is negative. Whether the market size increases or decreases in $k$ depends on the relative strength of the two effects.

Using (4), (8) and (11) yields the equilibrium number of open source users as

$$
X_o^*(k) = \frac{v_o(v_o(v_c - v_o + \psi) - kv_c) - 2\psi v_c(v_o - k)}{2(v_o - k)(v_o(v_c - v_o) - kv_c)}
$$

(15)

The effect of $k$ on this demand depends on $\psi$. To determine how, I define three thresholds. The first threshold is $\psi_1$ below

$$
\psi_1 = \{ \psi : \frac{\partial X_o^*}{\partial k} \bigr|_{k=0}(\psi) = 0 \} = \frac{v_o(v_c - v_o)^2}{v_c + (v_c - v_o)^2}
$$

(16)

The threshold $\psi_1$ is the only value of $\psi$ at which the marginal effect of the first unit of contributions has no impact on the number of open source users. The second threshold is $\psi_2 = \frac{v_o(v_c - v_o)}{2v_c}$. This is the value of $\psi$ for which $\psi = k(\psi)$ where $k$ is defined in lemma 2. For $\psi < \psi_2$ both products enjoy a positive market share for any value of $k$ satisfying the assumptions of the model. For $\psi > \psi_2$ there exist a sufficiently large $k$ for which the market share of the open source is zero.

The third threshold is $\psi_3 = \frac{v_o(v_c - v_o)}{2v_c}$. According to lemma 2, the firm always sets price $P_{H}^I$ when $k > k(\psi)$; so its product is alone in the market. Thus, there are two software products in the market only if $k(\psi) > 0$ because for $k > k(\psi)$ the firm sets price $P_{H}^I$. With such a price the size of the market for open source is zero (see lemma 2). Because $k(\psi)$ is decreasing in $\psi$ on the relevant interval, $k(\psi) > 0$ if, and only if,
Figure 2: Threshold $k$ as a function of cost $\psi$. The three regions represent the parameter range that results in duopolistic equilibria and that is acceptable under the model’s assumptions.

$\psi < \frac{v_o(v_c - v_o)}{2v_c - v_o}$.\(^{13}\)

Figure 2 shows the ranking of thresholds and displays $k$ as a function of $\psi$. Regions I, II, and III shows the combinations of $\psi$ and $k$ for which both software products are in the market.

In regions I and II, the $45^\circ$ line lies below $k(\psi)$. In these regions, the largest $k$ satisfying assumption (2) is smaller than $k(\psi)$. Therefore, the firm never sets price $p_a^H$. In these regions, the cost of learning about open source software is low, and the closed source product cannot be priced so as to keep the open source product out of the market.

In region III, there exists a $k$ large enough so that the equilibrium price is $p_a^H$, and the open source has a zero market share. It remains to be shown that the number of open source users may increase or decrease in $k$.

The following lemma yields a condition that I use in later proofs.

**Lemma 3.** Jointly $v_c > v_o$ and condition $\psi < \psi_3$ imply $v_o > 2\psi$.

**Proof.** Rearranging $\psi < \psi_3$, I obtain $2\psi(v_c - \frac{v_o}{2}) < v_o(v_c - v_o)$. Because $v_c > v_o$, it

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\(^{13}\)Note that if $\psi$ is larger than $\psi_3$ then $k < 0$ and if so by virtue of lemma 2 the open source market share is zero. In addition, assumption (2) ensures that the ranking of the thresholds is always $\psi_1 < \psi_2 < \psi_3$, and it follows from (16) that $\psi_1 > 0$.\]
must be true that $v_o > 2\psi$.

The following proposition characterizes how the equilibrium number of open source users evolves as a function of contributions.

**Proposition 1.** a) If $\psi \geq \psi_1$, then $X^*_o(k)$ is concave and strictly decreasing in $k$. b) If $\psi < \psi_1$, $X^*_o(k)$ always increases in $k$ when contributions are close to zero. c) Furthermore, if $\psi < \psi_1$ and $\psi$ is sufficiently small, then $X^*_o(k)$ increases even when contributions are significant.

*(Proof in Appendix.)*

In Region I, the number of open source users may or may not increase in contributions. The consumers’ expectations effect is strong because the cost associated with open source software is low. The price effect by contrast is weak because contributions are low. The reason is that price is not sensitive to variations in contributions (recall that $\frac{\partial^2 P}{\partial k^2} < 0$). Although the price decreases in $k$, the number of open source users can increase in $k$. As contributions increase, more consumers with a low valuation of quality opt for the open source software.

In region II, the price effect is strong enough to ensure that the size of the open source market decreases monotonically in contributions. Though, it is not strong enough to price the open source software out of the market. Therefore, even at the region’s boundary (at $k = \psi$), the market is shared by both products.

In region III, the price effect dominates the consumers’ expectation effect. The number of open source users decreases in $k$. Note that proposition 1 is true only when assumption (2) holds true. When it does not the hedonic cost can be negative. In such case, increases in $k$ would eventually lead to capture of the entire market by the open source software. Indeed, if the hedonic cost is negative, the open source software captures consumers with a high valuation of quality.

### 3.1.1 The welfare effect of open source entry

I can now examine how entry of the open source product into a market initially served by a closed source product affects the number of users and welfare.
Proposition 2. The price of the closed source product is lower and its market larger under duopoly than under closed source monopoly.\textsuperscript{14}

Proof. There are two parts to the proof. Part 1 concerns the price and part 2 concerns the market size.

Part 1. On inspection of the competitive price \( p_{cH}(0) \), it follows that \( p_{cH}(0) < p_m \) (\( p_m \) is given in subsection 2.1). Because \( p_{cH} \) is strictly decreasing in \( k \), this inequality holds for all \( k \).

Part 2. Upon substitution of \( p_{cH} \), given by (11), into (10), I obtain the number of closed source users in the duopoly as
\[
X^*_c = 1 - \tilde{\sigma} = 1 - \frac{1}{2} + \frac{\psi v_o}{2 (v_o - v_c - kv_c v_o)}.
\]

Because the market size is \( 1/2 \) under a monopoly, it follows from assumption (9) that the market size of the closed source software is larger when there also is an open source software in the market. When the firm sets price \( P = p_{aH} \), its market size is \( 1 - \frac{\psi}{v_o} \) which is larger than \( 1/2 \) (see lemma 3). I conclude that the market size is always larger under competition than under a closed source monopoly. \( \square \)

It follows from proposition 2 that entry of the open source product increases the surplus of all consumers who purchased software prior to entry. Clearly, consumers, who do not purchase the closed source product prior to entry, do so post entry and also gain. This group includes users of the open source software and additional users of the closed source product. The addition of this group contributes positively to total welfare.

3.1.2 The welfare effect of closed source entry

I now examine how entry of the closed source product into a market initially served by an open source product affects the number of users and welfare.

\textsuperscript{14}In the context of a horizontally differentiated market of Hotelling type, Zacharias [2009] finds that although competition may add to variety, it also leads to increase in prices which may harm the consumer welfare. Here, with vertical differentiation, the price of the closed source software is always decreased with entry.
Proposition 3. The number of software users is smaller under a duopoly than under an open source monopoly.\footnote{This is in contrasts to one of Klemperer’s results, in a model in which a competitive market faces entry from another firm which may be more cost efficient than the incumbent, he finds that increased competition always increases the industry output [Klemperer, 1988]. Here, the opposite is observed. Despite the entrant being more efficient, the increased competition has lowered the market size relative to the market size the open source software would enjoy if it were alone in the market.}

Proof. As \( X_o^* + X_c^* = 1 - \sigma_o^* \), I can simply insert \( X_o^* \) given by (15) into (3) to obtain

\[
1 - \sigma_o^* = X_o^* + X_c^* = 1 - k^2 v_c + 2(v_c - v_o)v_o \psi + k(v_o(v_o + \psi) - v_c(v_o + 2\psi)) \over 2(v_o - k)(v_o(v_c - v_o) - kv_c) \tag{17}
\]

This expression gives the size of the market served by both type of software. It follows from (17) and (5) that the total number of software users is smaller than under the open source monopoly if, and only if,

\[
- k ((v_o(v_c - v_o) - kv_c) + v_o \psi) \over (v_o - k)(v_o(v_c - v_o) - kv_c) < 0.
\]

This expression is indeed negative as assumption (9) along with assumption (2) entails that both the numerator and the denominator are positive. \qed

Competition lowers contributions to open source software, which lowers the value of the open source software. Because the open source software serves consumers with a low valuation of quality, some consumers who chose the open source software under the monopoly are no longer interested in using that software. Consequently, the total number of software users declines.

Consider the effect of closed source entry on consumer welfare. One may have expected such entry to increase welfare of all consumers because all consumers retain the option of using open source software. The following numerical example shows that this is not true.

Figure 3 shows consumers’ surplus as a function of \( \sigma \) under monopoly and duopoly for the particular case where \( v_c = 4, v_o = 1, \psi = 8/25, \) and \( k = 7/27 \). The indices \( \sigma_M \) and \( \sigma_D \) denote the marginal consumer under the monopoly and duopoly. The open source monopoly serves consumers with \( \sigma \) belonging to the interval \([\sigma_M, 1]\) where
\(\sigma_M = 1/18\). Under duopoly users belong to the interval \([\sigma_D, 1]\) where \(\sigma_D = 5/18\).

The consumer who has the same surplus under monopoly and duopoly is indexed \(\sigma_I = \frac{v_o(v_c - v_o) - kv_c + v_o(2k - \psi)}{2(v_c - v_o)(v_o - k)}\).

Consumers with index \(\sigma < \sigma_I\) lose surplus upon entry. Those with \(\sigma \in [\sigma_D, \tilde{\sigma}]\) continue to use the open source software, but lose because that software is less valuable to them as a result of the decline in contributions. Those with \(\sigma \in [\sigma_M, \sigma_D]\) stop using software altogether. Consumers with \(\sigma \in [\tilde{\sigma}, \sigma_I]\) switch to the closed source product but derive less surplus from it than they got from the open source product prior to entry. Finally, those with \(\sigma > \sigma_I\) use the closed source product. There surplus is larger than under an open source monopoly because the closed source software is of higher quality than the open source software. Note that \(\sigma_I \in (\tilde{\sigma}, 1/2)\).

The gains of consumers who benefit from entry are larger than the losses of those who are affected adversely by it. To see why, refer again to figure 3. Choose a point

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**Figure 3:** Closed source entry: comparison of the consumer’s surplus when \(v_c = 4, v_o = 1, \psi = 8/25, k = 7/25\)

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such that the distance between $\bar{\sigma}$ and $\sigma_I$ equals the distance between $\bar{\sigma}$ and $\bar{\sigma}'$.

The area of the triangle abc equals the area of the triangle cde. This means that the aggregate change in welfare for consumers on the interval $[\bar{\sigma}, \bar{\sigma}']$ is zero. It remains to be shown that the area degf is larger than the area $\sigma_Mba\sigma_D$. Note that within area degf, the consumer with the smallest gain from entry is consumer indexed $\bar{\sigma}'$. That consumer’s gain equals the loss of the consumer with the largest loss, which is the consumer indexed $\bar{\sigma}$. There are more gainers than losers as $\sigma_I < 1/2$. Thus, aggregate gains exceed aggregate losses. As a result, consumer welfare increases with entry. Total welfare which includes profits must therefore also increase.\(^{17}\)

### 3.2 Perfect discrimination

I now assume that the firm knows the preferences of each consumer. To simplify the analysis, I separate the consumers into two groups\(^{18}\): a group $L$ composed of individuals with $\sigma \in [0, \sigma_o)$, and a group $H$ composed of individuals with $[\sigma_o, 1]$ where $\sigma_o$ is defined by (3).

For consumers in group $L$, the best alternative to the closed source software is the outside option, which gives zero surplus. Hence, the firm sets the price to consumer $\sigma$ as $P_L(\sigma) = \sigma v_c$. This is the highest price that induces consumers in the group $L$ to buy. Since consumers in group $H$ obtain a positive surplus from the open source software, the firm must leave these consumers with a positive surplus in order to persuade them to buy. The price $P_H(\sigma) = \sigma (v_c - v_o) + \psi$ is the highest price that induces consumers in the group $H$ to buy the closed source software.\(^{19}\)

\(^{17}\)This is somewhat similar to Klemperer’s result that entry of an efficient competitor may be socially detrimental in the sense that some consumers lose [Klemperer, 1988]. Note that Klemperer’s model has products that differs in cost, but my model has products that differ in quality. Zacharias [2009] also shows that the entry of a second firm in a Hotelling type market may harm consumers as prices increase and consumers’ surplus potentially decreases.

\(^{18}\)By virtue of assumption (2) these two groups always exist.

\(^{19}\)Note that at these prices, if the fulfilled-expectations condition is satisfied, the consumers expect a market of size zero for the open source software.
Specifically, the discriminatory price schedule is

\[
P(\sigma) = \begin{cases} 
\sigma v_c & \text{for } \sigma \in [0, \psi/v_o] \\
\sigma(v_c - v_o) + \psi & \text{for } \sigma \in (\psi/v_o, 1] 
\end{cases}
\]

(18)

Figure 4 displays the price schedule (18). This price schedule entails zero usage of the open source software. However, the mere existence of the open source software compels the firm to leave positive surplus to consumers in group \( H \).

3.2.1 The welfare effect of open source entry

The entry of an open source software into a market initially served by a closed source monopoly leaves total welfare unchanged, but increases consumer surplus at the expense of profits. The reason is that the presence of the open source software forces the firm to leave a positive surplus to some consumers, those who obtain a positive surplus from an open source software that has an expected market size of zero.

3.2.2 The welfare effect of closed source entry

By contrast, the entry of closed source software into a market served by open source software increases total welfare, but consumers’ welfare falls. The reason is that the market is not covered under the open source monopoly, but is fully covered when the closed source firm engages in perfect discrimination.

Recall that the market is never covered under an open source monopoly and that the market is covered when the firm price discriminates. Since \( v_c > v_o \), the surplus increases faster in \( \sigma \) when consumers use the closed source software. Since all consumers use the closed source software, the surplus associated with each consumer is
higher than the surplus obtained from the opens source software. Clearly, the total surplus is higher after the entry of the closed source software. However, the consumer surplus is reduced. The consumers who did not consume software prior to entry are unaffected by entry. They consume the closed source software, but the firm charges them a price equal to the size of their respective surplus. As a result, these consumers are unaffected by entry. The consumers who used the open source software prior to entry lose from entry. They receive either zero surplus from the closed source software or a surplus equal to the surplus they would receive if the open source software had zero contribution. The latter is clearly lower than the surplus they received under the open source monopoly (since the software did receive contributions). Thus, consumers have a surplus either equal or lower than the surplus they received prior to entry.
4 Duopoly: Low-quality closed source software

I now study a market where the closed source software is of lower quality than the open source software. That is $v_c < v_o$. Again, I look at two equilibria: one with uniform pricing, and one with price discrimination.

4.1 Uniform pricing

In an equilibrium in which both products are consumed, consumers are separated into three groups:

- those with indices $\sigma < \sigma_c$ do not use software,

- those with indices $\sigma \in [\sigma_c, \tilde{\sigma}]$ purchase the closed source software, and

- those with indices $\sigma > \tilde{\sigma}$ choose the open source software.

To address the effect of contributions, I first derive the price chosen by the firm. As in section 3.1, the firm can choose a positive price which keeps the open source software out of the market. This price is $P^L_a = v_c + \psi - v_o$.

Unlike the case where the closed source software is of higher quality (see lemma 1), $P^L_a$ may be negative. When the closed source software is of high quality, all consumers prefer the closed to the open source software when the former’s price is zero. When the open source software is of high quality, for certain values of the parameter, the only price that can keep the open source out of the market is negative.

The duopoly price is

$$P^L_c = \frac{v_c(\psi - k)}{2(v_o - k)}$$ (19)

As in case where the closed source software is of higher quality, there is a threshold for parameter $k$ beyond which the firm sets a price sufficiently low to keep the open source software out of the market. When $k > \bar{k}(\psi) = \frac{2v_o(v_c - v_o + \psi) - v_c \psi}{v_c - 2(v_o - \psi)}$, the firm sets the price $P^L_a$. 
I now investigate the effect of consumers’ contributions on the number of open source users. The equilibrium number of open source users is given by

\[ X_o(P^L_c(k)) = \frac{P^L_c(k) + v_o - v_c - \psi}{v_o - v_c - k} \]  

which upon substitution for \( P^L_c(k) \) from (19) yields

\[ X_o^*(k) = \frac{2(v_o - k)(v_o - \psi) - v_c(2v_o - \psi - k)}{2(v_o - k)(v_o - v_c - k)} \]

As for the case where the closed source software is of higher quality than the open source software, there is a consumers’ expectations effect and a price effect. Since price \( P^L_c \) decreases in \( k \) and since (20) increases in price, the price effect of \( k \) is to lower the number of open source users. On the other hand, the direct effect of \( k \) captured via the denominator increases the number of open source users.

Before determining the net effect of a change in \( k \), I must ensure that the parameters are such that the open source software enjoys a positive market share. I define three thresholds for \( \psi \).\(^{20}\) The first threshold is \( \psi_1 = v_o - v_c \). The value \( v_o - v_c \) is the fixed point of the \( \overline{k} \) function, viz. \( \psi_1 = \overline{k}(\psi_1) \). The second threshold is

\[ \psi_2 = \frac{v_o((v_o - v_c)^2 + v_o(v_o - v_c))}{v_o^2} \]

This threshold is the value of \( \psi \) at which the marginal effect of the first unit of contributions has no impact on the number of open source users, viz. \( \psi_2 = \{ \psi : \frac{dX_o}{dk} \bigg|_{k=0} = 0 \} \).

The third threshold is \( \psi_3 = \frac{2v_o(v_o - v_c)}{2v_o - v_c} \). As before, I impose \( \overline{k}(\psi) > 0 \) which is a necessary condition in order to have an equilibrium where both software products have a positive market share. In equilibrium, there are two software products in the market only if

\[ \psi < \frac{2v_o(v_o - v_c)}{2v_o - v_c} = \psi_3 \]  

(21)

These thresholds determine the three regions shown in figure 5. In region I, where \( \psi < \psi_1 \) and \( k < \psi \), the cost of learning about open source is small, making the open source software a tough competitor to the closed source software. Consequently, there is no \( k \) large enough to allow the firm to price the open source software out of the

\(^{20}\) These thresholds have the same interpretation as in section (3.1), although their values differ.
In region II, where \( \psi \in [\psi_1, \psi_2] \), the firm prices the open source software out of the market when the contributions to open source software are significant, that is, when \( k \) is large. In region III, where \( \psi > \psi_2 \), the threshold function \( \overline{k}(\psi) \) is lower than in region II. The closed source firm prices the open source out of the market at lower values of \( k \).

The following proposition gives some properties of the equilibrium number of open source users.

**Proposition 4.** If \( \psi \leq \psi_1 \), the function \( X_o^* \) is convex and increasing in \( k \). If \( \psi > \psi_1 \), the function \( X_o^* \) is decreasing for \( k \) sufficiently large. For \( \psi > \psi_1 \), I do not show whether the function is concave or convex.

*(Proof in appendix.)*

In region I, the closed source market size decreases and the open source market size increases in \( k \). The reason is that when \( k \) is large the low value of \( \psi \) weakens the firm’s ability to capture consumers. In that region, the open source software enjoys the benefits from increases in contributions. As the hedonic cost \( \phi = kX_o^e - \psi \) goes
to zero, even the consumers with a low valuation of quality prefer the open source software. Consequently, the closed source software is squeezed out of the market. The firm’s reaction to the increase in $k$ is to lower its price in order to capture consumers with low valuation of quality. However, even for a low price, these consumers prefer the open source software when the hedonic cost goes to zero. This contrasts with the result found in subsection 3.1 where at low values of $\psi$, the firm sells software to more than half the market (see proposition 2).

In region II, the effect of contributions on the number of open source users depends on the magnitude of $k$. If $k$ is small, the effect on price is weak and the user contributions effect dominates the price effect. Consequently, the number of open source users increases in $k$. However, if $k$ is sufficiently large, the effect on price becomes more important and user contributions effect is dominated by the price effect. Thus, when $k$ is sufficiently large, the number of open source users decreases in $k$. In region III, the number of open source users is decreasing in $k$. In this region, the price effect dominates the consumers’ expectations effect for all values of $k$. Consequently, in this region, the number of open source users always decreases in $k$.

In summary, the size of the market for closed source software decreases in $k$ in region I but increases in $k$ in regions II and III. Thus, if demand for closed source software increases in $k$ at any level of contribution, then there exists a level of contribution which ensures that the open source software has a zero market share. By contrast, if demand for closed source software decreases in $k$ at any level of contribution, then there exist a level of contribution at which the closed source software has a zero market share.

4.1.1 The welfare effect of open source entry

I now consider the welfare effect of entry by an open source software in a market initially served by a closed source firm.

Consider first consumers who purchased the closed source software prior to entry and continue to do so post-entry. These consumers clearly gain from entry as the post-entry price of the closed source software is lower than the pre-entry price. The
consumers who switch from closed source to the open source software also gain because they would have gained if they did not switch. Consumers who did not purchase prior to entry and use either the closed source or the open source product post entry also gain. The remaining consumers do not use any software before and after entry are unaffected. Thus, no consumer loses from entry while some gain.

4.1.2 The welfare effect of closed source entry

The total size of the market is larger under a duopoly than under an open source monopoly. The total number of software users under duopoly is $1 - \sigma_c = 1 - \frac{\psi - k}{2 v_o - k}$, whereas the number of open source users in under monopoly is $1 - \sigma_o = 1 - \frac{\psi - k}{v_o - k}$. Note that in contrast with the result in proposition 3, entry now increases the total number of software users. This is explained by the fact that consumers with a high valuation of quality use software regardless of entry. When the firm is a low-quality entrant, it gets most of its market from consumers who, before entry, did not use software. Furthermore, in the monopoly case as contributions increases the market is eventually covered. However, under a duopoly it may not be covered even when $k$ is large. Note that under the monopoly $X_o \to 1$ as $k \to \psi$, but under the duopoly $X_c + X_o \to 1 - \frac{v_o - \psi}{v_c} < 1$ as $k \to \bar{k} < \psi$ and remains constant for $k \in [\bar{k}, \psi)$.

I now consider welfare. Figure 6(a) contrasts the consumer’s surplus before and after entry for the particular case where $v_c = 2$, $v_o = 1$, $\psi = 1.1$, and $k = 0.6$. These values are within region II of figure 5. The consumer indexed $\sigma = \sigma_I$ has the same

Figure 6: Comparison of the consumer’s surplus when $v_o = 2$, $v_c = 1$, and $\psi = 1.1$
surplus whether or not the closed source software enters the market. Consumers with $\sigma > \sigma_I$ lose from entry, and those with $\sigma < \sigma_I$ gain. The loss incurred by consumers with $\sigma > \sigma_I$ increases in $k$, whereas the gain of consumers with $\sigma < \sigma_I$ decreases in $k$. Thus, aggregate consumer surplus falls with entry when the contributions are significant.

Figure 6(b) looks at consumers’ surplus as a function of $k$ when $v_c = 2$, $v_o = 1$, and $\psi = 1.1$. The values of $k$ consistent with duopoly belong to the interval $[0, 1.1]$. Aggregate consumer surplus falls with entry when $k > k_I$, where $k_I$ is defined as the value of $k$ at which the aggregate consumer surplus under monopoly equals the surplus under duopoly. The aggregate consumer surplus under monopoly and duopoly aggregate surplus increases in $k$. However, the surplus under duopoly increases less rapidly than under monopoly. The monopoly surplus increases in $k$ because, as $k$ increases, more consumers use the open source software and more surplus accrues to those already using it.

Recall that an increase in $k$ decreases the price of the closed source software. This decrease in price has two effects: 1) It induces consumers with a low valuation of quality to buy the closed source software; 2) it induces consumers who would otherwise choose the open source software to choose the closed source software. The second effect may lower the value of open source software because less consumers are contributing. As a result, open source consumers may not benefit from the increase in contributions.

This result, in terms of welfare, is similar to the result of Casadesus-Masanell and Ghemawat [2006] who found that the monopoly outcome may dominate the duopoly outcome. However, it contrasts with their results in that in their model the entry of the open source Linux induces some consumers to switch from Windows to Linux. In my model, the loss in efficiency is caused by the switch from the open source to the closed source. The reason for the difference is that in Casadesus-Masanell and Ghemawat [2006] the closed source product has more value, while in this section I assumed that the open source software has more value.
4.2 Price discrimination

I now assume that the firm knows consumers’ willingness to pay. The consumer indifferent between both products has index

$$\sigma = \frac{\phi - P(\sigma)}{v_o - v_c}.$$

I assume that the firm can offer negative prices to some consumers, which is equivalent to sponsoring a consumer to use the software.\textsuperscript{21}

The firm’s maximization problem is then

$$\Pi = \max_{P(\sigma), \sigma} \int_0^{\sigma(X^e_o)} P(\sigma) d\sigma$$

subject to

$$(IR) \sigma v_c - P(\sigma) \geq 0 \text{ for all } \sigma \in [0, \sigma(X^e_o)]$$

$$(IC) \sigma v_c - P(\sigma) \geq \sigma v_o - \phi \text{ for all } \sigma \in [0, \sigma(X^e_o)]$$

$$X^e_o = 1 - \sigma(X^e_o)$$

The last equality in (22) is the fulfilled-expectations condition. In group $L$, defined in subsection 3.2, the consumer indexed $\sigma$ buys the closed source software if, and only if,

$$\sigma v_c \geq P(\sigma) \ \forall \ \sigma \in [0, \sigma_o).$$

The firm charges the consumer indexed $\sigma$ a price $P_L(\sigma) = \sigma v_c$. A consumer in group $H$ buys the closed source software if, and only if, $S_c(\sigma) = \sigma v_c - P(\sigma) \geq \sigma v_o - \phi = S_o(\sigma, X^e_o) \ \forall \ \sigma \in [\sigma_o, \sigma]$. By collecting the $\sigma$ terms and rearranging, the latter condition becomes $\phi - \sigma(v_o - v_c) \geq P(\sigma) \ \forall \ \sigma \in [\sigma_o, \sigma]$. In group $H$, the firm charges consumer indexed $\sigma$ a price $P_H(\sigma) = \phi - \sigma(v_o - v_c) \ \forall \ \sigma \in [\sigma_o, \sigma]$. The optimal price function\textsuperscript{22}

\textsuperscript{21}Consumer $\sigma$ gets paid to use the closed source software. In practice, firms sponsor influential and sophisticated consumers. Firms sponsor universities by providing them with research centers and free software. The negative price found in the model is equivalent to a real-life situation in which a firm sponsors some influential consumer. For example, a software firm may sponsor a university department because it gains from precluding the department from contributing to the quality of an open source software, for example, GEDCO, a Canadian company, helped the Ionian University of Greece with a sponsorship valued at €250 000 for geophysical software.

\textsuperscript{22}Note that $P'_H(\sigma) < 0$ and $P'_e(\sigma) > 0$. 
is therefore

\[
P(\sigma) = \begin{cases} 
P_L(\sigma) = \sigma v_c & \text{for } \sigma \in [0, \sigma_o) \\
P_H(\sigma) = \phi - \sigma(v_o - v_c) & \text{for } \sigma \in [\sigma_o, \bar{\sigma}] \\
\infty & \text{otherwise} 
\end{cases}
\] (23)

Because the firm does not sell to consumers with indices \( \sigma > \bar{\sigma} \), these consumers choose the open source software. All consumers use software, that is the market is covered. Figure 7 illustrates this; the price function is represented by the bold line.\(^{23}\)

The firm benefits from charging a negative price to some consumers belonging in group \( H \). The firm does so in order to reduce contributions thereby allowing higher prices for the closed source software.

Inserting the price schedule defined in (23) into the profit function defined in (22) allows me to write the firm’s maximization problem as

\[
\max_{\sigma} \Pi = \int_{0}^{\sigma_o(X_o^e)} P_L(\sigma) d\sigma + \int_{\sigma_o(X_o^e)}^{\sigma_o(X_o^e)} P_H(\sigma) d\sigma 
\] (24)

subject to

\[X_o^e = 1 - \bar{\sigma}(X_o^e) \]  

\(\text{Fulfilled expectations}\)

\(^{23}\)An interesting similarity with the present paper, is in the findings of Thisse and Vives [1988] of a price schedule which decreases in the distance to the firm. In their Hotelling model, the firm lowers its price because the competition is fiercer in remote places. In the present paper, I find that the firm’s price schedule may decrease over a certain range. Although my model uses vertical differentiation, the explanation for the decreasing price schedule is somewhat the same. The relation between Hotelling-type models of horizontal differentiation, and models of vertical differentiation is shown in Cremer and Thisse [1991].
Profit (24) can be rewritten as

\[ \Pi = \int_0^\sigma \sigma v_o d\sigma + \phi(X_o^e) \int_{\sigma_o(X_o^e)}^\sigma d\sigma - v_o \int_{\sigma_o(X_o^e)}^\sigma \sigma d\sigma. \]  

(25)

The first term of this expression is the surplus of group \( L \) which is entirely appropriated by the firm. The second term is the hedonic cost borne by group \( H \); and the third term is the total utility that group \( H \) gets from the open source product. Note that \( \sigma_o \) and \( \phi \) are both functions of \( X_e^o \), which in turn depends on \( \overline{\sigma} \). Differentiation of (25) with respect to \( \overline{\sigma} \) and collection of terms yields the first-order condition

\[ \frac{d\sigma_o}{d\overline{\sigma}} (v_o \sigma_o - \phi) + k(\overline{\sigma} - \sigma_o) + (\phi - \overline{\sigma}(v_o - v_c)) = 0. \]  

(26)

Since the first term of this expression is zero because it is the surplus of consumer \( \sigma_o \) and by definition of \( \sigma_o \) that surplus is zero. Hence,

\[ -k(\overline{\sigma} - \sigma_o) = (\phi - \overline{\sigma}(v_o - v_c)) = P_H(\overline{\sigma}). \]  

(27)

The firm profits from selling to all consumers in group \( L \). Therefore, the firm chooses \( \overline{\sigma} \) among the consumers in group \( H \). This means that \( \sigma_o < \overline{\sigma} \) which implies that the right-hand side of (27) is positive. This shows that in equilibrium the firm always charges a negative price to the consumer indexed \( \overline{\sigma} \).

4.2.1 The welfare effect of open source entry

In the closed source monopoly the market is covered and all consumers receive zero surplus. For consumers in group \( L \), entry as no effect. The profit associated with

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\(^{24}\)Bhaskar and To [2004] using a Hotelling-Salop model find that, with a fixed number of firms, perfect price discrimination provides incentives for firms to choose product characteristics in a socially optimal way, but that, with free entry, the number of firms is always excessive. In my model, when the firm sells at a negative price to some consumers, it does not directly create inefficiency as it is simply a transfer of surplus from the firm to the consumers. However, it does create inefficiency by lowering contributions to the open source software.

\(^{25}\)In an equilibrium where both software products are consumed, the consumer with index \( \sigma = \overline{\sigma} \) is indifferent between the open source and the closed source software, and is located at \( \overline{\sigma} = \frac{v_o - v_c - k}{v_o - v_c - k + v_c}. \) If \( k \) is large enough, then the firm sells to all consumers. Using the fact that \( X_o = 1 - \overline{\sigma} \), it follows that \( X_o > 0 \) if, and only if, \( k < \frac{v_c \psi - v_o \psi}{v_o - \psi} \). Thus, the number of open source users is positive when the level of contributions is low.
these consumers is the same and these consumers still have zero surplus. However, all consumers in group $H$ gain from entry. Either they consume a high quality open source software or they consume the closed source software from which they also obtain a positive surplus. Although the firm’s profit is decreased, the gains in consumer surplus outweigh the loss. Total welfare increases.

### 4.2.2 The welfare effect of closed source entry

Consider entry of a closed source software. The total surplus curve shown in panel (a) of figure 8 has a discontinuity at $\sigma$ because the profit associated with consumer indexed $\sigma = \overline{\sigma}$ is negative. Recall that the firm pays the consumer indexed $\sigma = \overline{\sigma}$ to use the closed source software. Consumers with indices $\sigma > \overline{\sigma}$ use the open source software.\(^{26}\) Overall the total welfare increases or decreases depending on the relative on the size of $k$.\(^{27}\)

In panel (b) of figure 8, I compare the total surplus of the monopoly with that of the duopoly when $v_o = 4$, $v_c = 1$, and $\psi = 2$. Panel (a) displays the total surplus associated with each consumer in $[0, 1]$ when $k = 1$. The surplus associated with each

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\(^{26}\)Stole [2007] presents an oligopoly game of first-degree price discrimination in which firm differs with respect to their cost functions. He mentions that while the effect on consumer welfare depends on the shape of the particular price function used, the price discrimination always increases total welfare. I will show that in the context of this paper, the entry of a closed source firm practicing first-degree price discrimination may reduce total welfare.

\(^{27}\)This makes a point in the ongoing argument between Nalebuff [2009] and Elhauge [2009] where the latter claims that one should not suppose that the total welfare effects of price discrimination are positive.
consumer indexed $\sigma < \sigma_I$ is higher under duopoly than under monopoly. Conversely, the surplus associated with each consumer indexed $\sigma > \sigma_I$ is lower under duopoly than under monopoly. Panel (b) compares the total surplus under duopoly and monopoly as a function of $k \in [0, \bar{k}]$. The total surplus is higher under monopoly when $k > k_I$ where $k_I$ is the value of $k$ at which the aggregate surplus under monopoly equals aggregate surplus under duopoly. The intuition is the same as with uniform pricing. When contributions are significant, the loss in value of the open source software is significant and this causes a loss for consumers of the open source software.
5 Conclusion

This paper addresses three questions: 1) how does the closed source firm pricing strategy affect the industry market structure; 2) how does entry of either closed or open source software affect welfare; and 3) how is the outcome to the aforementioned questions affected by price discrimination.

To address these questions, I establish a baseline by looking at a single closed source software firm first, and then at a single open source software producer. As in Mussa and Rosen [1978], the firm serves half the market in my closed source baseline. My second baseline has a market size that depends on user contributions.

In a competitive environment, the effect of user contributions has important consequences on the closed source firm’s strategy. The impact of this effect on the market draws upon the relative quality of the products and the pricing policy of the closed source firm.

The model suggests that user contributions can be detrimental to open source software by indirectly lowering its value. In fact, user contributions may divert consumers towards closed source software; indeed, eradication from the market is particularly a threat when user contributions are significant. If the closed source firm was indifferent to changes in the level of contributions the open source software would actually gain market share as contributions increase, but the firm’s aggressive pricing is detrimental the open source.

An increase in user contributions has two effects on the value of open source software. The first effect is direct, and adds value to the open source software through an increase in user contributions. The second is indirect, and counteracts the first effect. The closed source firm reacts by lowering its price thus lessening the open source’s value. As more consumers redirect to the closed source product the loss in value for the open source is compounded by the decrease in the number of open source users, therefore contributors.

The relative magnitude of these two effects ultimately determines whether user contributions increases or decreases the value of open source software. Higher levels of user contributions exacerbate the second effect. It amplifies the decrease in price
associated with variation in contributions. Consequently, when user contributions are significant, the second effect dominates the first one. The potential gains from increase in contributions are offset by the firm’s price decrease. When the closed source firm does react to changes in contributions, its aggressive pricing strategy forces the open source software out of the market. Hence, when the contributions to open source software are significant, the contributions appear as a competitive disadvantage for the open source software.

What do these various scenarios imply for welfare? To answer this question, I look at the case of an open source software targeting consumers who have a high valuation of quality. Such a case results in a lesser value for the open source software given the introduction of a closed source firm targeting consumers who have a low valuation of quality. The lessened value of the open source software engenders a loss for consumers with a high valuation of quality, and this loss is greater when the level of user contributions is high. As indicated, the loss in value associated with a decrease in the size of the market for open source increases with the level of users contributions. The loss is even more important when the firm practices first-degree price discrimination. The firm’s motive is as before, it sells at low prices to consumers with a high valuation of quality — it sells at a negative price to some consumers — to stifle the effect of user contributions. Consumers with a low valuation of quality may gain from entry because they may purchase the low quality closed source software. Overall, such entry, regardless of the entrant’s pricing technique, may reduce total welfare.

While entry may be detrimental to welfare, it would be natural to assume that entry would at least increase industry output. This paper provides evidence to the contrary. It shows that entry of a closed source software serving consumers with a high valuation of quality lowers the number of software users. As previously noted, the entry reduces the value of the open source software. Therefore, some users with a low valuation of quality abandon the open source even though they were deriving benefit prior to entry.

Economic models of software markets tend to assume that the marginal production cost is zero. I have adopted this approach. However, whether the quality of closed source software is higher or lower than that of open source software could be made endogenous by assuming that the cost of quality is positive. In my model, the fate of the
open source software is entirely determined by the decision of the closed source firm. This may well be critical to the conclusion that the open source product may be eradicated from the market. It would be of interest to examine whether such an outcome is possible if the strategy of the open source's managing entity were to maximize its market size. Also, the model assumes that only two software products are competing. One could introduce another competitive closed source firm. There are other potential extensions to the model: a more thorough examination of the indirect network effect (the user contributions) driving the results of the model; the firm could choose its location; a direct network effect can be introduced so that compatibility issues are studied. Another vantage point could consider a dynamic approach where a software benefits from a first-mover advantage or an existing user base.
6 Appendix

Proof of proposition 1. I proceed to the analysis of the function, $X^*_o$, for the three cases (shown in figure 2) in which the parameter $\psi$ is in region I, II, and III, respectively.

The derivative of $X^*_o$ evaluated at $k = 0$ is $\frac{\partial X^*_o}{\partial k}|_{k=0} = \frac{\psi_0 - \psi}{2v_o} - \frac{\psi^2}{2(v_c-v_o)v_o^2}$ which, by definition of $f^H$ (see equation (16)), vanishes at $\psi = f^H$. Because the expression is strictly decreasing for $\psi > 0$, it follows that it is positive for $\psi < f^H$ and negative for $\psi > f^H$. Thus, the function $X^*_o$, when evaluated at $k = 0$, is increasing in region I; and decreasing in region II and III.

In region I, the parameter $k$ is bounded by $\psi$ (see assumption (2)), and thus, I evaluate the derivative at its boundary $k = \psi$, which yields $\frac{\partial X^*_o}{\partial k}|_{k=\psi} = \frac{\psi_0 v_o}{2(\psi_0 - \psi)^2} + \frac{\psi_0^2 (v_c - v_0)(\psi - \psi_0)}{2(\psi_0 - \psi)(v_c - v_0)^2}$. The denominators are equal and positive, so I focus only on the numerators. The first term (numerator) is negative for $\psi < \psi_o \left(1 - \frac{\psi_0}{2v_c}\right)$, but $\psi$ is always smaller than this value because I am interested in cases where $\psi < \overline{\psi}$; and $\psi < \psi_o \left(1 - \frac{\psi_0}{2v_c}\right)$. The second term is positive and decreasing in $\psi$ (lemma 3 states that $\psi_0 - 2\psi > 0$). Thus, when the absolute value of the first term is greater than the value of the second term, the expression is negative; and the converse is also true. Because the first term is linear in $\psi$ and the second term quadratic, the line describing the absolute value of the firm term can cross the line described by the second term at most twice. In fact, the two terms crosses at points: $\psi_L = \psi_o \frac{3\psi_0 - 2v_0}{4v_c} - \frac{\psi_0 \sqrt{\psi_0^2 - 4v_c(v_c - v_0)}}{4v_c}$ and $\psi_H = \psi_o \frac{3\psi_0 - 2v_0}{4v_c} + \frac{\psi_0 \sqrt{\psi_0^2 - 4v_c(v_c - v_0)}}{4v_c}$. I reject $\psi_H$ because $\psi_H > \overline{\psi}$. Thus in the relevant range for $\psi$, the absolute value of the first term crosses the line described by the second term only once at $\psi_L$. Now, two properties ensures that the derivative of $X_o$, when evaluated at $k = \psi$, is increasing in $k$ for $\psi < \psi_L$ and decreasing in $k$ for $\psi > \psi_L$: both terms are decreasing, and the second term is convex in $\psi$. In region I, $\psi < f^H$, and so $0 < \psi_L < f^H$ entails that, depending on the value of the parameters, the function can be both decreasing and increasing in contributions. The demonstration also shows that $X^*_o$ is decreasing in region II at point $k = \psi$ because $\psi_L < f^H < \psi$.

In region II and III, the demand function is strictly concave in $k$: I need to show
that $\frac{\partial X_o^2}{\partial k^2} < 0$ where
\[
\frac{\partial X_o^2}{\partial k^2} = \frac{v_o}{(v_o - k)^3} \frac{v_o(v_c - v_o) - kv_c + v_c(v_o - k)}{(v_o - k)^3(v_o(v_c - v_o) - kv_c)^3} \\
\left(-k\psi v_c \frac{v_o(v_c - v_o) - kv_c + v_c(v_o - k)}{(v_o - k)^3(v_o(v_c - v_o) - kv_c)^3} + \psi v_o^2 \frac{(v_c - v_o)^2 - v_o v_c}{(v_o - k)^3(v_o(v_c - v_o) - kv_c)^3}\right).
\]

To simplify the function, I multiply it by $(v_o - k)^3(v_o(v_c - v_o) - kv_c)^3$; and that does not change the sign of the function because, by assumption (9), $v_o(v_c - v_o) - kv_c$ is positive. Then, the sign of $\frac{\partial X_o^2}{\partial k^2}$ is as the sign of
\[
v_o \frac{v_o(v_c - v_o) - kv_c}{(v_o - k)^3} - \frac{v_o(v_c - v_o) - kv_c + v_c(v_o - k)}{(v_o - k)^3(v_o(v_c - v_o) - kv_c)^3} \\
\left(-kv_c \frac{v_o(v_c - v_o) - kv_c + v_c(v_o - k)}{(v_o - k)^3(v_o(v_c - v_o) - kv_c)^3} + v_o^2 \frac{(v_c - v_o)^2 + v_c v_o}{(v_o - k)^3(v_o(v_c - v_o) - kv_c)^3}\right) \psi.
\]

I now show that this expression is decreasing in $\psi$ and use $\psi = f^H$ to find its least upper bound with respect to $\psi$. The first term does not affect $\psi$, the second term — before “×” — is positive because of assumption (9), and it follows from assumption $v_c > v_o$ and assumption (2) that the third term is positive. Evaluating the expression at $\psi = f^H$ yield
\[
-v_o^2 k^2 v_c \frac{k(2v_c - v_o) - 3v_o(v_c - v_o)) + (v_c - v_o)^2 v_o}{(v_c - v_o)^2 + v_c^2}
\]
which constitutes a least upper bound for the expression since $\psi > f^H$. Now, the resulting expression is increasing in $k$. To see this, note that the maximum acceptable $k$ is the fixed point where $\bar{k}(\psi) = \frac{v_o(v_c - v_o)}{2v_c} = \psi$. Now, since $k < \frac{v_o(v_c - v_o)}{2v_c} < \frac{3v_o(v_c - v_o)}{2v_c - v_o}$ it follows that $k(2v_c - v_o) - 3v_o(v_c - v_o) < 0$ in the relevant range. Because the expression is increasing in $k$, evaluating the function at $k = \frac{v_o(v_c - v_o)}{2v_c}$ (the maximum acceptable value for $k$), again, constitutes a least upper bound. The expression evaluated at $k = \frac{v_o(v_c - v_o)}{2v_c}$ is reduced to $-v_o^2 \frac{(v_c - v_o)^2}{(2v_c - v_o)^2}$ which is strictly negative. I conclude that $X_o^*$ is strictly concave for $\psi > f^H$.

As for the slope of the function, because $X_o$ is strictly concave, together $\frac{\partial X_o^*}{\partial k} \bigg|_{k=k_1} < 0$ and $\frac{\partial X_o^*}{\partial k} \bigg|_{k=k_2} < 0$ imply $\frac{\partial X_o^*}{\partial k} < 0 \forall k \in [k_1, k_2]$ where $k_1 < k_2$. Thus, it suffices to show
that the function is strictly decreasing when evaluated at the boundaries of the relevant parameter space in order to prove that the market size is strictly decreasing.

I already showed that $X_o^*$, when evaluated at $k = 0$, is decreasing in regions II and III. And I showed that it is decreasing at the boundary $k = \psi$ in region II. It remains to be shown that the function is decreasing at the upper bound of region III.

In region III, $\psi > \bar{k}$ and thus the maximum value that $k$ can take is $k = \bar{k}$ at which point $\frac{\partial X_o^*}{\partial k} \bigg|_{k=\bar{k}} = -\frac{v_o^2(v_o-2\psi)^3}{2v_o^2(v_o-\psi)^3} < 0$ (by virtue of lemma 3, the numerator is positive).

The following lemma will be used in the proof of the next proposition.

**Lemma 4.** Together assumption 2, condition (21), and assumption $k < \bar{k}$ imply that $\psi_1 = v_o - v_c > k$.

The lemma simply states that if both software products are in the market in equilibrium, the difference in quality must be large enough relative to contributions.

**Proof of proposition 4.** I proceed to the analysis of the function, $X_o^*$, for the three cases where the parameter $\psi$ is in region I, II, and III, respectively.

First, I show that $X_o^*$ is strictly convex in $k$ when $\psi$ is on the interval $[0, v_o - v_c)$, that is in region I. The second-order derivative of $X_o^*$ with respect to $k$ can be written as $\frac{\partial X_o^*}{\partial k^2} = A + B\psi$, where

$$B = \frac{(2v_o - v_c - 2k)(k^2 + v_c^2 + v_ck + v_o(v_o - v_c - k))}{(v_o - k)^3(v_o - v_c - k)^3}.$$ 

It follows from corollary 4 that $B$ is negative since both the numerator and the denominator are positive. Thus, the second-order derivative is decreasing in $\psi$. Since $\psi < v_o - v_c$, evaluating the expression at $\psi = v_o - v_c$ constitutes a lower bound. This lower bound $\frac{\partial X_o^*}{\partial k^2} \bigg|_{\psi=v_o-v_c} = \frac{v_c}{(v_o-k)^3}$ is positive implying that the second-order derivative is negative for all $\psi \leq v_o - v_c$. I conclude that $X_o^*$ is strictly convex when $\psi < v_o - v_c$.

I now look at the slope of the function. At $k = 0$ the derivative of $X_o^*$ evaluates to

$$\frac{\partial X_o^*}{\partial k} \bigg|_{k=0} = \frac{v_o((v_o - v_c)^2 + v_o(v_o - v_c))}{2(v_o - v_c)^2v_o^2} - \frac{(v_o - v_c)^2 + v_o^2}{2(v_o - v_c)^2v_o} \psi.$$

...
The fact that the expression is strictly decreasing in $\psi$, and the fact that the expression vanishes at $\psi = f^L$ entails that the expression is positive for $\psi < f^L$. Thus, the demand is increasing at $k = 0$ in region I and II, and decreasing at $k = 0$ in region III.

In region I, since I showed that the function is strictly convex and that it is strictly increasing at $k = 0$, it suffices to show that the function is also increasing at $k = \psi$ to prove the first statement of the proposition. The derivative of $X_o^*$ evaluated at $k = \psi$ yields

$$\frac{\partial X_o^*}{\partial k} \bigg|_{k = \psi} \frac{v_o - v_c - \psi + (v_o - \psi)}{2(v_o - \psi)(v_o - v_c - \psi)} > 0 \text{ since } \psi < v_o - v_c.$$ 

In region II and III, I evaluate the function at $k$ yielding

$$\frac{\partial X_o^*}{\partial k} \bigg|_{k = \bar{k}} = \frac{(v_o - v_c - \psi + (v_o - \psi))^3}{2v_c(v_o - \psi)(v_o - v_c - \psi)}.$$ 

The expression has a negative denominator since $\psi > v_o - v_c$. The numerator is positive since $\psi < \Psi_L < \frac{2v_o - v_c}{2}$, so the expression is negative. □
References


