Tax Competition, Imperfect Capital Mobility and the gain from non-preferential agreements.*

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Abstract

The gain to competing governments from entering into binding non-preferential tax agreements (that prevents discriminatory taxation in favor of mobile capital) depends on the extent of capital mobility between jurisdictions. In particular the gain is increasing in the cost of relocation of capital and the fraction of domestic tax base that is relatively immobile. We show this in a symmetric model of capital tax competition between two governments where all capital is imperfectly mobile and differ in their cost of relocation.

JEL classification: F15; F21; H26; H87

Keywords: Tax competition; Capital mobility; Non-preferential regime

1 Introduction

Fiscal competition between governments of sovereign nations and other jurisdictions (including regional and local governments) is an important determinant of economic policy and the state of public finance across the world. An important concern of public economics and political economy relates to the fact that tax competition may reduce tax revenues, erode tax bases and cause other harmful effects through inefficient allocation of resources across space.¹ One particular aspect of this problem that has attracted considerable attention in recent years (among economists as well as policy makers) is the widespread use of preferential capital tax rates to attract mobile capital from other jurisdictions. Globalization and the removal of barriers to capital mobility has made relocation of capital easier over time and this has increased the incentives for national and other governments to engage in aggressive preferential capital taxation that can be direct (countries give

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¹See Wilson (1999) for a review of tax competition literature.
tax subsidy to foreign investors) or indirect\(^2\) (countries set low tax rate on investment in activities that attract large foreign capital).

In recent years, concerned by the perceived "harmful effects" of such preferential measures adopted competitively by large number of countries, several international agreements and non-binding resolutions have been adopted by the European Union\(^3\) (EU)\(^4\) and Organization for Economic Co-operation and Development (OECD)\(^5\) in order to impose restrictions on preferential taxation among member countries and to take joint action against continuation of preferential tax regimes by non-member countries. The primary "harmful effect" motivating such agreements appears to be the erosion of tax revenue, rather than economic efficiency. However, apart from the EU and OECD, there does not appear to be any other demand for imposition of restrictions on preferential capital taxation\(^6\), despite widespread policy cooperation and coordination between blocks of nations on other issues such as trade barriers and monetary policy. It appears, therefore, that the economic gains from multilateral restriction on preferential capital taxation may not be always be positive and may depend on the economic fundamentals of the competing nations. We investigate the economic foundations of non-preferential capital tax agreements. In particular, we analyze the strategic incentives and gains from such agreements in the context of capital tax competition and relate them to the composition and degree of mobility of capital bases.

This paper is a contribution to the significant theoretical literature that has focused on the comparison of tax revenues generated under capital tax competition with and without restrictions on preferential taxation. Keen (2001) analyzes a symmetric game of tax competition between two countries that compete over two exogenous capital bases and shows that if the elasticity of investment flow with respect to tax differential is not too high, then tax revenues generated in Nash equilibrium are actually higher under preferential taxation (where the two countries set discriminatory taxes on the two bases) relative to non-preferential taxation (when the countries are required to set the same tax rate on the two bases). Non-preferential regimes distort tax rates (as optimal tax rates are different for capital bases with different elasticity) and spread competition for more elastic investment to less elastic investment, resulting in lower total tax revenues. Wilson (2005)

\(^{2}\)It is important to notice that a preferred treatment of foreign residents is often granted indirectly rather than directly. Take the famous case of Ireland, which levied only a 10% tax rate on corporate income in the manufacturing and financial service sectors instead of the standard rate (32%).

\(^{3}\)Main emphasis in meeting of Council of Economics and Finance Ministers (1997) was to formalize "Design to detect such tax measures which unduly affect the location of business activity in the Community by being targeted merely at non-residents and by providing them with a more favorable tax treatment than that which is generally available in the Member State concerned. In 1998 EU Group established to identify harmful tax measures. By Nov 1999, Group identified 66 harmful tax measures".

\(^{4}\)The European Commission’s (1997) “code of conduct on Business Taxation” is a non-binding resolution among members state to avoid preferential taxation of certain activities including foreign investment.

\(^{5}\)In 1998, the OECD adopted its "Guideline on Harmful Preferential Tax Regimes" (see, OECD, 1998). OECD (2000) Committee on Harmful Tax Practices identified 47 preferential tax regimes. In its progress report, OECD (2004) mentions that, 18 of these abolished, 14 amended and 13 not found to be harmful on further analysis. OECD (2006) report states "The Committee considers that this part of the project has fully achieved its initial aims. Future work in this area will focus on monitoring any continuing and newly introduced preferential tax regimes identified by member countries".

\(^{6}\)Indeed, by the year 2007, there were only fourteen bilateral agreements for the exchange of information for tax purposes (which is absolutely essential to monitoring and identifying preferential regimes).
finds similar result when one of the two capital bases is perfectly mobile. These theoretical results then lead to the natural question: why do countries enter into non-preferential tax agreements? The literature identifies three factors that may explain this - "home bias", endogenous size of capital base and asymmetry.

Haupt and Peters (2005) introduce "home bias" in the Keen (2001) model by assuming that each country has a natural advantage in one of the two capital tax bases in the sense that when both countries set equal tax rates for that base, one country gets a larger fraction of the investment from that base. Under preferential regime, each country is more aggressive and offers excessive tax discount to the base in which it has a disadvantage which reduces total revenues of both countries. In contrast, when each country chooses the same tax rate for the aggregate of the two tax bases, neither has any incentive to be more aggressive than the other so that total revenues may be higher.

Janeba and Smart (2003) consider a generalized version of Keen’s model where the size of the tax base is not exogenous but depends on the tax rate. Under preferential regime, the tax rate on the more elastic tax base is lower and therefore the tax base itself is larger. Restrictions on preferential taxation increases the tax rate on the more elastic tax base (the larger base under preferential taxation) and decreases the tax rate on the less elastic capital base (which is smaller in size under preferential taxation). Therefore, starting from a preferential regime, a small restriction on preferential taxation increases tax revenue. A complete ban on preferential taxation is desirable if the resultant increase in the tax rate on more elastic capital bases does not erode the base significantly.

If countries differ significantly in size and one of the capital base is perfectly mobile, it has been shown that non-preferential tax regimes can generate higher tax revenue. In a model with two tax bases, one of which consists of perfectly immobile investors and other of perfectly mobile investors, Janeba and Peters (1999) argue that while under preferential regime, countries engage in Bertrand tax competition that reduces tax on mobile capital to zero, under non-preferential regime, the country with smaller captive segment of investors (immobile capital) is more aggressive in tax competition and focuses on attracting mobile capital while the country with larger captive segment is content with earning high tax revenue from immobile segment. The smaller country (country with smaller immobile capital base) earns higher revenue under non-preferential regime while the other country’s tax revenue remains exactly the same as under preferential regime. Emphasizing the role of asymmetry in obtaining this result, Wilson (2005) shows that when countries are symmetric, preferential and non-preferential regimes generate equal tax revenues.

Unlike asymmetry in size and composition of capital bases between countries, asymmetry in productivity of capital between two countries may actually increase the appeal of preferential taxation if the smaller country is also less productive. Nicolas, Steeve and Wilson (2007) argue that when one of the capital bases is perfectly mobile and the other is perfectly immobile, non-preferential regime causes the smaller country to be more aggressive in reducing taxes (smaller domestic immobile capital segment) and if the smaller country is also less productive, then it may lead to a reduction in the total output and hence the joint tax revenues of the competing countries.


This result may change if small country is also less productive.

The introduction of asymmetry between competing jurisdictions does not necessarily overturn Keen’s result on
In this paper, we focus on the effect of differences in the extent of capital mobility on the comparison of tax revenues generated under preferential and non-preferential regime. In particular, we consider a framework where there is no asymmetry between the countries and the tax competition game is fully symmetric, there is no home bias (in the sense of Haupt and Peters, 2005) and the total size of the tax bases are exogenously fixed (unaffected by the tax rates). We show that nonetheless, non-preferential tax regime may generate higher tax revenue if capital is not extremely mobile.

We analyze a symmetric model with two countries (jurisdictions) where the governments engage in capital tax competition in order to maximize their tax revenue. Each country has a unit mass of capital owners that differ only in their capital mobility and total investment is exogenously fixed. A fraction of capital owners in each country is perfectly immobile while the other fraction is partially mobile in the sense that investors have to incur a fixed cost of capital relocation to invest in the other country. The model generalizes the analysis in Wilson (2005) and Marceau, Mongrain, Wilson (2007) to allow mobile capital owners to be imperfectly mobile. All mobile capital owners are assumed to be identical - this allows us to treat the cost of relocation as a single parameter. The degree of capital mobility in our model is captured by the fraction of investors in each country that are mobile and the fixed cost of relocation of mobile investors.

Under preferential regime, governments can set different tax rates for mobile and immobile investors that locate in their jurisdiction (independent of their origin). Under non-preferential regime, each government sets the same tax rate on all investment made within its jurisdiction. We compare the Nash equilibrium tax revenue outcomes between the preferential and non-preferential regime and characterize how this comparison depends on the degree of capital mobility.

There are three main contributions of this paper. First, we provide an economic foundation on the basis of degree of capital mobility for existing non-preferential tax agreements in Europe and OECD countries and for the fact that there are large parts of the world where governments do not appear to be moving towards any such tax agreement. In particular, we argue that a combination of the cost of capital relocation and composition of tax bases in terms of capital mobility may provide some understanding of this phenomenon. In this respect, our analysis complements the existing literature that has focused on other factors such as asymmetry, home bias and effect of tax rate on the size of tax base. In particular, we provide a clear characterization of the comparative statics of capital mobility on the tax revenue gains from switching to a non-preferential tax agreement. Second, we characterize mixed strategy equilibria for a class of Bertrand tax/price competition games where competitors need to undercut rivals by an exogenous discrete amount in order to steal business or attract investors from rival jurisdiction. Note that the existing literature on models of homogenous good price competition with captive consumer segments\(^\text{11}\) as well as tax competition with perfectly immobile segments, assumes that non-captive consumers or mobile investors always

\(^{11}\) Among many see for example; Butters (1977), Shilony (1977), Rosenthal (1980), Varian (1980), Deneckere et al. (1992), Butters (1977), Narasimhan (1988), Roy (2000),
move to the firm or the country with lower price/tax\textsuperscript{12} i.e., there is no fixed cost or relocation or preference for one product over another. The mixed strategy equilibria characterized in our paper should be useful to a much larger literature in applied game theory and industrial organization. Finally, in the equilibrium of our model, though all mobile capital owners are identical and countries set different tax rates with probability one, both countries receive a positive share of investment from mobile capital owners i.e., the highly elastic tax base.\textsuperscript{13}

The remainder of the paper has the following structure. Section 2 introduces the model of tax competition. Sections 3 and 4 characterize the Nash equilibrium of the tax competition game under non-preferential regime and preferential regime, respectively. Section 5 compares the tax revenues generated under the two regimes. Section 6 concludes.

2 Model

We consider two symmetric countries, labeled as country 1 and country 2 respectively. Each country consists of homogenous individuals and aggregate capital owned by residents of each country are fixed\textsuperscript{14} and normalized to unity. Fraction $\lambda$ of total capital\textsuperscript{15} in each country is internationally mobile (though imperfectly) and fraction $1 - \lambda$ of capital is perfectly immobile, where $0 < \lambda < 1$. If an individual in country $i$ wants to invest in country $j$ where $i \neq j$, he has to incur an additional exogenous fix cost\textsuperscript{16} of investment abroad denoted as $F \geq 0$. Parameter $F$ captures barrier to international mobility of capital. Government of country $i$ receives tax only from total investment in country\textsuperscript{17} $i$. Governments maximize tax revenue\textsuperscript{18} and capital owners maximize net after tax returns on capital. Productivity of capital is same in both countries\textsuperscript{19} and for simplicity we assume that investors get the portion of their capital not taxed. Thus investment decisions of individuals


\textsuperscript{13}In contrast to Wilson (2005), Janeba and Peters (1999), Nicolas, Steeve and Wilson (2007)) In a CES Lecture Course on Theories of Tax Competition, Wilson emphasizes that the next step in research on multiple tax bases is, “Build a model with an equilibrium where both countries obtain some of a highly elastic base. (Does this model exist?) Are preferential or non-preferential regimes desirable?”.

\textsuperscript{14}Such restrictions seems reasonable if we think of tax bases as capital income in different sectors or industries, where effective tax rate may differ due to different depreciation rules or other sector-specific tax provisions. Spillovers are likely to play a bigger role when tax bases represent real and financial capital in one sector, or capital and labor income more generally.

\textsuperscript{15}We have assumed total investable capital and total capital to be same.

\textsuperscript{16}Addition cost can also be considered as cost of attaining information regarding investment opportunity in foreign countries. This model can incorporate the scenario when government can tax foreign-source income. In this case additional cost will include the cost of investors moving abroad themselves to avoid domestic taxation.

\textsuperscript{17}Taxing foreign source income can be extremely difficult and costly for governments. As noted by Janeba and Smart (2005) and also by Wilson (1999), taxing foreign-source income implies severe administrative and tax compliance problems. But even if it could be enforced in practice, this would not eliminate tax competition. In this case, investors are themselves are still able to move abroad to avoid domestic taxation.

\textsuperscript{18}This is not the same as assuming that the government is of Leviathan type (see Edwards and keen,1996, for a discussion of the Leviathan hypothesis).

\textsuperscript{19}Departing from the standard assumption of a declining marginal product of capital is frequent in the literature and simplifies our analysis. For papers which investigate the case in which capital tends to agglomerate because of an increasing marginal product, see Baldwin and Krugman (2004), Boadway, Cuff and Marceau (2004), Kind, Knarvik and Schjelderup (2000).
are only guided by tax rate set by two countries. Maximum tax rate a country can impose on any form of capital is equal to 1. To see how cost of capital relocation affects individuals investment decision, suppose \( \tau^i \) and \( \tau^j \) are tax rates set by country \( i \) and country \( j \) on mobile capital. An investor in country \( i \) invests his mobile capital in country \( i \) as long as \( |\tau^i - \tau^j| < F \), and he invests in country \( j \neq i \) if \( \tau^i \geq \tau^j + F \). We consider the nature of tax competition between two countries under “preferential regime” and “non-preferential regime”. Under non-preferential regime, countries pre-commit to uniform tax rate for both mobile and immobile capital, whereas under preferential regime countries can set different tax rates on mobile and immobile capital. Throughout this paper we have assumed that countries do not charge different tax rate on domestic and foreign capital. The equilibrium concept is Nash equilibrium of tax competition. Below we formally describe the nature and timing of the game under preferential regime and non-preferential regime.

Preferential regime: The game consists of three stages. Stage one: each country simultaneously announces tax rates on immobile and mobile capital. Stage two: capital owners decide whether to locate their capital in country 1 or country 2 observing tax rates set by two countries. Stage three: countries receive tax payments from total investments in respective countries.

Non-preferential regime: Second and third stage of the game is exactly similar to the case of preferential regime. Stage one: each country pre-commits to uniform tax rate for mobile capital and immobile capital. Countries announce tax rate simultaneously.

3 Non-Preferential Regime

In this section we analyze the nature of tax competition when governments can not set different tax rates on mobile capital and immobile capital. Extreme case when a fraction of capital is perfectly mobile (\( F = 0 \)) has been analyzed by Wilson (2005) and Janeba and Peters (1999). But intermediary case when capital is imperfectly mobile has not been analyzed.

Strategy space of country \( i \) is defined as \( \tau^i \in [0, 1], \ i \in \{1, 2\} \). Let \( \tau^1 \) and \( \tau^2 \) be tax rates set by respective governments of country 1 and country 2. Payoff function of country \( i \) (which is also gross tax revenue of country \( i \)) for strategy pair \( (\tau^i, \tau^j) \) can be described as,

\[
R^i (\tau^i, \tau^j) = \begin{cases} 
\tau^i, & \text{if } |\tau^i - \tau^j| < F \\
\tau^i + \lambda (\tau^i), & \text{if } \tau^i \leq \tau^j - F \\
(1 - \lambda) \tau^i, & \text{if } \tau^i \geq \tau^j + F 
\end{cases}
\]

where \( i, j = \{1, 2\} \) and \( i \neq j \). From (1) we can see that tax revenue of a country is discontinuous. Discontinuity occurs when \( |\tau^i - \tau^j| = F \).

Pure strategy Nash equilibrium does not exist if \( F > \left(1 + \frac{1}{\lambda}\right)^{-1} \). A country’s tax revenue is not a concave function of its tax rate. Each country may desire a high tax rate when other country’s tax rate is low, while also desiring a low tax rate when the other country’s tax rate is high. While complete description of equilibrium is provided in the appendix, below we describe the nature of mixed strategy Nash equilibrium in more detail.
Mixed strategy Nash equilibrium under such circumstance has not been analyzed. Dasgupta and Maskin (1986) provide example where a firm can sell to all consumers by undercutting price charged by competing firm by small margin, which results in mixed strategy Nash equilibrium with *atomless convex* support. Varian (1980) characterizes symmetric mixed strategy Nash equilibrium of a much general price competition model with \( n \) firms and free entry. Unlike Varian’s model, the case when there are only two countries/firms with no entry which may allow countries/firms earn positive tax revenue/profit is also considered in Wilson (2005), Wang (2005) and Narasimhan (1988) among others. We analyse the case when countries/firms have to undercut competing countries/firms by discrete amount to attract mobile capital/consumers. This generalization is significant as it provides tools to study many issues such as labor/capital movement due to tax/environmental differences and also when countries have immobile capital base which is our current exercise. Outcome will be similar when firms compete with differentiated products but they also have captive segment. Limitation of this equilibrium is that two countries have equal immobile capital base and we do not find equilibrium when cost of capital mobility is very small. Finding mixed strategy Nash equilibrium with unequal immobile capital base/captive segment and relaxing the constraint we imposes below provide further challenge. We make following assumption to insure that equilibrium of tax competition between countries under non-preferential regime is well defined.

**Assumption (1):** We impose following restriction below to insure that equilibrium of tax competition under non-preferential regime is well defined.

\[
F \geq (1 - \lambda) \left( \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right)^{-1} \equiv \Upsilon
\]

Assumption (1) states the constraint on \( \lambda \) and \( F \) which insure that equilibrium of tax competition under non-preferential regime is well defined. If we fix \( \lambda \) then \( F \) should be at least as large as certain critical value which satisfy (2) with equality. This assumption is also explained later in figure (4).

We only consider symmetric mixed strategy Nash equilibrium. Hence we remove superscript to identify individual countries. To find mixed strategy Nash equilibrium we consider \( G(.) \) the tax distribution (mixed strategy) both countries use in equilibrium. Subsequently \( sop (dG) \) denotes the support of mixed strategy Nash equilibrium. Given tax revenue of a country for any tax pair \( (\tau^i, \tau^j) \) is given by (1), expected tax revenue of a country \( (E(R)) \) in equilibrium can be written as,

\[
E(R) = \tau \left( 1 - \lambda \right) + \tau \lambda \left[ 1 - G(\tau - F) \right] + \tau \lambda \left[ 1 - G(\tau + F) \right] \forall \tau \in sop (dG).
\]

For any tax rate \( \tau \) a country always receives tax from its immobile capital base. With probability \( [1 - G(\tau - F)] \) it retains domestic mobile capital and with probability \( [1 - G(\tau + F)] \) it also attracts mobile capital from competing country. Below in proposition (1) we describe the outcome of tax competition under non-preferential regime formally.
Proposition (1): If \( F \geq (1 + \frac{1}{\lambda})^{-1} \) then pure strategy Nash equilibrium exists. In equilibrium both countries set tax rate 1. Both countries earn tax revenue equal to 1 in equilibrium. If \( F < (1 + \frac{1}{\lambda})^{-1} \), pure strategy Nash equilibrium does not exist. However mixed strategy Nash equilibrium exists under assumption (1).

(I) If \( \left(1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right)^{-1} < F < \left(1 + \frac{1}{\lambda}\right)^{-1} \) in symmetric mixed strategy Nash equilibrium both countries earn tax revenue equal to \( 1 - \Psi \) where,

\[
\Psi = \frac{1}{2} (1 + \lambda) - \frac{F}{2} - \frac{1}{2} \sqrt{F^2 + 2F(1 + \lambda) + (1 - \lambda)^2}
\]  

(II) If \( \Psi \leq F \leq \left(1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right)^{-1} \) in symmetric mixed strategy Nash equilibrium both countries earn tax revenue equal to \( \Delta \equiv \frac{F}{\lambda} \left(1 + \sqrt{1 + \lambda^2}\right) \).

(III) If \( F = 0 \) then pure strategy Nash equilibrium does not exist. In symmetric mixed strategy Nash equilibrium countries earn tax revenue equal to \( (1 - \lambda) \).

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As shown in fig (1) in symmetric mixed strategy Nash equilibrium countries randomize over the interval \( \{(1 - \Psi - F, 1 - F), (1 - \Psi, 1)\} \). Noting that \( 0 < \Psi < F \) in mixed strategy Nash equilibrium no country sets tax rate in the range \( (1 - F, 1 - \Psi) \). This is possible because probability distribution of tax rates country \( i \) uses in equilibrium is \textit{continuous} everywhere on the support with probability mass of nonzero measure at 1. Say country \( i \) \( (i = 1, 2) \) sets high tax rate (greater than \( 1 - \Psi \)) then country \( j \) \( (j \neq i, j = 1, 2) \) undercuts by the margin of \( F \) to attract

\[
0 < \Psi < F < (1 + \frac{1}{\lambda})^{-1} \Rightarrow 0 < \Psi < F. \text{ It is easy to check that } F < (1 + \frac{1}{\lambda})^{-1} \Rightarrow \Psi > 0.
\]

Further \( \Psi < F \) if \( f(F) < 0 \) where \( f(F) = 4\lambda - 8F(1 + \lambda) + 8F^2 \). Now \( f''(F) = 8 > 0 \) and \( f'(F) < 0 \) if \( F < \frac{1 + \lambda}{2} \), which implies \( f'(F) < 0 \) when \( F < (1 + \frac{1}{\lambda})^{-1} \). Thus maximum of \( f(F) \) occurs at \( \left(1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right)^{-1} \). At \( \left(1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right)^{-1} = F \) we have \( f(F) = 0 \). Noting that \( f(F) \) is continuous, inequality is obvious.
mobile capital. But when $F$ is sufficiently high country $i$ does better by just lowering its tax rate by small amount so that it can retain its domestic mobile capital. If a country sets tax rate equal to $1 - \Psi$ it is no longer revenue maximizing for competing country to undercut. If a country sets tax rate in the range $(1 - \Psi, 1)$ its tax revenue depends on tax distribution of competing country in the range $(1 - \Psi - F, 1 - F)$ and similarly for tax rate in the range $(1 - \Psi - F, 1 - F)$, tax revenue depends on tax distribution over the range $(1 - \Psi, 1)$. Let $G^1(.)$ and $G^2(.)$ be probability distributions of tax rate used by countries in equilibrium over intervals $(1 - \Psi - F, 1 - F)$ and $(1 - \Psi, 1)$ respectively. Expected tax revenue of competing countries $E[R(\tau)]$ in equilibrium can be written as,

$$E[R(\tau)] = \begin{cases} (1 - \lambda) \tau + \lambda \tau + \lambda [1 - G^2(\tau + F)], & for \tau \in (1 - \Psi - F, 1 - F) \\ (1 - \lambda) \tau + \lambda [1 - G^1(\tau - F)], & for \tau \in (1 - \Psi, 1) \end{cases}$$

(5)

If a country sets tax rate equal to $1 - \Psi$ it retains its mobile capital with probability one and it does not attract mobile capital from competing country. Thus tax revenue of competing country in equilibrium is $1 - \Psi$. We can find $\Psi$ considering some $G^1(.)$ and $G^2(.)$ and comparing the tax revenue of a country at tax rate 1 and $1 - \Psi - F$ respectively and using the fact that in mixed strategy Nash equilibrium countries earn equal tax revenue everywhere on the support and because there is probability mass except at tax rate 1 we also have $G^1(1 - \Psi) = G^2(1 - F)$. Once we know infimum of the support we can find equilibrium tax distribution using (5).

Figure 2

If $F$ is comparatively lower, i.e. $\Upsilon \leq F \leq \left(1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right)^{-1}$ mixed strategy Nash equilibrium takes of the form in fig (2). In equilibrium countries randomize over the interval $(\Delta - F$, 1].
\( \Delta + F \). Support of mixed strategy Nash equilibrium is convex and supremum of the support is strictly less then 1 if \( F < \left( 1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right)^{-1} \). There is no probability mass anywhere on the support. The intuition behind this equilibrium is similar to the previous one discussed above. If a country sets tax rate \( \Delta \) then it receives tax from its mobile and immobile capital with probability one. In this case no country is able to attract mobile capital from other country. Thus countries earn tax revenue of \( \Delta \) in equilibrium. Expected tax revenues of competing country can be described similar to (5), denoting \( G^1 (.) \) and \( G^2 (.) \) as probability distribution of tax rates used by countries in equilibrium over intervals \( (\Delta - F, \Delta) \) and \( (\Delta, \Delta + F) \) respectively. We find \( \Delta \) by comparing the tax revenue of a country for tax rate \( \Delta - F \) and \( \Delta + F \) and the fact that \( G^1 (\Delta) = G^2 (\Delta) \). Once we know \( \Delta \) we can find tax distribution by using similar method used in part (I).

Since irrespective of tax rate set by competing countries, a country can set tax rate equal to 1 and earn \((1 - \lambda)\) from immobile capital base, hence, in equilibrium expected tax revenue of competing countries should be at least \((1 - \lambda)\). But \( F < Y \) implies \( \Delta \equiv \left( \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right) < (1 - \lambda) \), where \( Y \) is given by (2). When \( F < Y \) undercutting by competing countries results in very low tax rate. A country does better by setting tax rate equal to 1 and forgo tax revenue from mobile capital completely. Nash equilibrium under this is not clear. Below we discuss few issues related to existence of Nash equilibrium in this symmetric tax game.

This game is not reciprocally upper semi-continuous, one of conditions for existence of mixed strategy Nash equilibrium described in Corollary 5.2 in Reny (1999). Below we provide the formal definition of reciprocally upper semi-continuous game as described in Reny (1999).

**Definition:** A game \( G = (X_i, u_i)_{i=1}^N \) is reciprocally upper semi-continuous if, whenever \((x, u)\) is in the closure of the graph of its vector payoff function and \( u_i(x) \leq u_i \) for every player \( i \), then \( u_i(x) = u_i \) for every player \( i \).

To see that symmetric tax game we analyze in this paper is not reciprocally upper semicontinuous, consider strategy pair \((F, 0)\) where country 1 and 2 sets tax rate equal to \( F \) and 0 respectively. For strategy pair \((F, 0)\) country 1 receives tax revenue equal to \( R^1(F, 0) \equiv (1 - \lambda)F \) and country 2 receives tax revenue equal to \( R^2(F, 0) \equiv 0 \). Now for tax strategy pair \((F - \varepsilon, 0)\) we have \( R^1(F - \varepsilon, 0) = F - \varepsilon \) and \( R^2(F - \varepsilon, 0) = 0 \). \( F - \varepsilon > (1 - \lambda)F \) for \( \lambda > 0 \) and \( \varepsilon < \lambda F \). Thus we have \( R^1(F - \varepsilon, 0) > R^1(F, 0) \) and \( R^2(F - \varepsilon, 0) = R^2(F, 0) \), contradicting that the game is reciprocally upper semicontinuous.

In a two players game reciprocally upper semicontinuity requires that whenever a player’s payoff jump up, payoff of other player jump down. In this game we provide a counter example where payoff of country 1 jump up but payoff of country 2 remains constant. Following Reny (1999) we formally describe the mixed extension of the game.

**Definition:** Assume that each \( X^i \) in this game is a compact Hausdorff space. Thus \( G = (X^i, u_i)_{i=1}^{i=2} \) is a compact, Hausdorff game. Consequently, if \( M^i \) denotes the set of (regular, countable additive) probability measure on the Borel subsets of \( X^i \), \( M^i \) is compact in the weak* topology. Extend each \( u^i \) to \( M = X_{i=1}^{i=2} M^i \) by defining \( u^i(\mu) = \int_X u^i(x) d\mu \) for all \( \mu \in M \). Then the game \( \hat{G} = (M, u^i)_{i=1}^{i=2} \) denotes the mixed extension of \( G \).
It is easy to check that our game is compact, Hausdorff game as 
\[ \tau^i \in [0, 1], \quad i = 1, 2. \] 
Thus mixed extension of this game is well defined. Now consider a mixed strategy in the mixed extension of our game where country 1 and 2 sets tax rate equal to \( F \) and 0 respectively with strictly positive probability. Because tax rates countries set are assumed to be non-negative, country 1 can increase its tax revenue by redistributing probability from tax rate \( F \) to some lower value \( F - \epsilon > 0 \). Hence we show that mixed extension of our game is not reciprocally upper semicontinuous.

Note that sum of payoffs of two countries described by (1) is not upper semicontinuous, one of the condition for existence of mixed strategy Nash equilibrium described in Dasgupta and Maskin (1986). Upper semicontinuity of sum of payoffs is stronger requirement than reciprocally upper semicontinuity. Still we show it formally below because this condition is so widely used to show existence of mixed strategy Nash equilibrium.

**Definition:** Suppose \( X \) is a topological space, \( x_0 \) is a point in \( X \) and \( f : X \rightarrow \mathbb{R} \) is a real-valued function. We say that \( f \) is upper semi-continuous at \( x_0 \) if for every \( \epsilon > 0 \) there exists a neighbourhood \( U \) of \( x_0 \) such that \( f(x) < f(x_0) + \epsilon \) for all \( x \) in \( U \). Equivalently, this can be described as,

\[
\lim_{x \to x_0} \sup_{x \in U} f(x) \leq f(x_0)
\]

Where \( \lim \sup \) is the limit superior (of function \( f \) at the point \( x_0 \)).

To see that \( \sum_{i=1}^{2} R^i (\tau^i, \tau^j) \) is not upper semi-continuous consider the tax pair \( (\tau + F - \epsilon, \tau) \) where \( \tau + F - \epsilon \) and \( \tau \) are tax rates set by country \( i \) and country \( j \) respectively. For all \( F > \epsilon > 0, \sum_{i=1}^{2} R^i (\tau^i, \tau^j) = 2\tau + F - \epsilon \). Similarly for tax pair \( (\tau + F + \epsilon, \tau), \sum_{i=1}^{2} R^i (\tau^i, \tau^j) = 2\tau + (1 - \lambda) F, \forall \epsilon \geq 0 \). Thus we have,

\[
\lim_{\epsilon \to 0^+} \sum_{i=1}^{2} R^i (\tau^i, \tau^j) = 2\tau + (1 - \lambda) F < 2\tau + F - \epsilon \quad \text{for} \quad \epsilon < F.
\]

When \( F = 0 \) both countries earn \( (1 - \lambda) \) in equilibrium. Support of mixed strategy Nash equilibrium is convex and there is no probability mass anywhere on the support. In equilibrium both countries randomize over the interval \( \left[ \left( \frac{1-\lambda}{1+\lambda} \right), 1 \right] \) as shown in fig (3). Competition for mobile capital drives down expected revenue from mobile capital to zero. See Varian (1980) for complete characterization of mixed strategy Nash equilibrium of similar nature.

Wilson (2005) also analyzes the case when \( F = 0 \). In this model as long as \( F > \Upsilon \) in equilibrium countries earn tax revenue which is strictly higher then \((1 - \lambda)\). The result differs due to the fact that, now a country has to undercut tax rate set by competing country by the margin of \( F \) to attract mobile capital. Thus imperfect capital mobility not only reduces gains from undercutting, it also reduces tax revenue from immobile tax base (cost of undercutting). These two effects together reduce competition, hence, increase tax revenue.

**Proposition (2):** Suppose \( F \geq \Upsilon \), under non-preferential regime country’s tax revenues decreases monotonically as \( F \) decreases or \( \lambda \) increases.
Result of proposition (2) is not surprising. If $F < (1 + \frac{1}{\lambda})^{-1}$ then pure strategy Nash equilibrium does not exist and countries set low tax rates on mobile and immobile capital to attract mobile capital from competing country. As $F$ decreases it becomes less costly for owners of mobile capital to relocate their capital in the country with lower tax rate. Similarly large $\lambda$ provides more incentive to countries to set lower tax rate to attract mobile capital from competing countries. Also loss of tax revenue from immobile base capital due to lower tax is small for large $\lambda$.

4 Preferential Regime

Under preferential regime countries set different tax rates for mobile and immobile capital. Now for given $F$ competition for mobile capital is more intense. This is because under preferential regime, fraction of capital being immobile will not have any effect on competition for mobile capital. Now irrespective of $F$, countries set maximum possible tax rate (equal to 1) on immobile capital base. Since fraction $(1 - \lambda)$ of total capital is immobile, both countries earn tax revenue equal to $(1 - \lambda)$ from their respective immobile capital base. Tax revenue a country earns from mobile capital depends on $F$, and tax rate set by other country on mobile capital. Let $\tau^i$ and $\tau^j$ be tax rate set on mobile capital by country $i$ and country $j$ respectively. Total tax revenue of of country $i$ can be described as,

$$R^i(i^j, \tau^j) = (1 - \lambda) + \begin{cases} 
\lambda \tau^i, & \text{if } |\tau^i - \tau^j| < F \\
2\lambda \tau^i, & \text{if } \tau^i \leq \tau^j - F \\
0, & \text{if } \tau^i \geq \tau^j + F.
\end{cases}$$

(6)

where $i, j = \{1, 2\}$ and $i \neq j$. If $|\tau^i - \tau^j| < F$ then owners of mobile capital invest their capital in the country of their residence. When $\tau^i \leq \tau^j - F$, owners of mobile capital in both countries invest their capital in country $i$. Since fraction $\lambda$ of capital is mobile, country $i$ earns tax revenue equal to $2\lambda \tau^i$ from tax on mobile capital. The outcome is opposite when $\tau^i \geq \tau^j + F$. In this case owners of mobile capital in country $i$ invest their capital in country $j$.

Because tax revenue from immobile capital base is fixed, tax competition for mobile capital base is similar to symmetric Bertrand price competition where a competitor has to undercut tax rate/price set by rival entity by discrete amount. If cost of undercutting is sufficiently small then pure strategy Nash equilibrium does not exist. A country desire low tax rate when tax rate set by competing country is high and also desire higher tax rate when tax rate set by competing country is low. Thus a country’s strategy is not monotone with respect to strategy of competing country. It is clear from above discussion that game described as (5) does not satisfy reciprocally upper semicontinuity, condition described in Reny (1999) for the existence for mixed strategy Nash equilibrium. In a game with two players reciprocally upper semicontinuity require payoff of a player to jump down if payoff of other player jumps up. Note that in our game if one country sets tax rate equal to zero then tax revenue of other country is zero from mobile capital base if it sets tax rate slightly higher then $F$ and it earns strictly positive tax revenue if it sets rate slightly below $F$, failing to satisfy requirements for reciprocally upper semi-continuity. We find mixed strategy Nash
equilibrium of this tax competition for mobile capital base by using method similar to proposition (1). Now Nash equilibrium is well defined for all values of $\lambda$ and $F$. Below in proposition (3) we describe the nature of tax competition under preferential regime formally.

**Proposition (3):** In equilibrium both countries will set tax rate equal to 1 on immobile capital. Since fraction $(1 - \lambda)$ of capital is immobile, countries earn tax revenues equal to $(1 - \lambda)$ from immobile capital base. Tax rate on mobile capital depends on $F$. If $F \geq 1/2$ pure strategy Nash equilibrium exists. In equilibrium both countries set tax rate equal to 1. Both countries earn tax revenue equal to $(1 - \lambda)$. If $0 < F < 1/2$, pure strategy Nash equilibrium does not exist. However mixed strategy Nash equilibrium exists.

(I) If $1/(2 + \sqrt{2}) < F < 1/2$, in mixed strategy Nash equilibrium both countries earn tax revenue equal to $\lambda(1 - \Phi)$. Where,

$$\Phi = 1 - \frac{F}{2} - \frac{1}{2}\sqrt{F^2 + 4F}. \quad (7)$$

(II) if $0 < F \leq 1/(2 + \sqrt{2})$, then in mixed strategy Nash equilibrium both countries earn tax revenue equal to $\lambda F (1 + \sqrt{2})$.

(III) If $F = 0$ pure strategy Nash equilibrium exists. In pure strategy Nash equilibrium both countries set tax rate equal to zero on mobile capital base.

Now governments compete head-to-head to attract mobile capital from competing country. In equilibrium tax rate on mobile capital depends only on maximum possible tax rate (which is equal to 1) and $F$. It is easy and intuitive to see that pure strategy Nash equilibrium exist if $F \geq 1/2$. Cost of moving capital is so high that investors do not find it profitable to invest abroad unless tax difference between two countries is large. Because there is no threat of mobile capital moving abroad, two countries are completely segmented, and both sets highest possible tax rate on mobile capital as well.

When $F < 1/2$ pure strategy Nash equilibrium does not exist. The reason for non-existence of pure strategy Nash equilibrium is clear from above discussion. Note that if we substitute $\lambda = 1$ in proposition (1) we get tax distribution on mobile capital countries use in mixed strategy Nash equilibrium under preferential regime.

When $1/(2 + \sqrt{2}) < F < 1/2$, in mixed strategy Nash equilibrium countries randomize over the interval $\{(1 - \Phi - F, 1 - F) , (1 - \Phi, 1)\}$. The equilibrium is as shown in fig (1), with $\Psi$ replaced by $\Phi$. Note that $\Phi < F$, hence in equilibrium no country sets tax rate in the interval $(1 - F, 1 - \Phi)$. This is possible because there is probability mass of nonzero measure at 1. If a country sets tax rate equal to $1 - \Phi$ then it retains domestic mobile capital with probability one and it does not attract mobile capital from competing country as $\Phi < F$. Thus in equilibrium both

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21 Note that under preferential regime countries sets tax rates for mobile and immobile capital independent of each other. Thus $\lambda$ does not play any role in competition for mobile capital. Putting $\lambda = 1$ in (2) we get $F \geq 0$, which is trivially satisfied.

22 $\Phi < F \Leftrightarrow f(F) = 8F^2 + 4 - 10F > 0$. Now $f(F)$ can be written as $8F(F - 1/4) + 8(1/2 - F)$. Hence $f(F) > 0$ as $1/(2 + \sqrt{2}) < F < 1/2$.
countries earn tax revenue equal to $1 - \Phi$. If $F \leq 1/(2 + \sqrt{2})$ in mixed strategy Nash equilibrium countries randomize over the interval $[\sqrt{2}F, F(2 + \sqrt{2})]$. The support of mixed strategy Nash equilibrium is convex and there is no probability mass anywhere on the support. The equilibrium is as shown in fig (2) with $\lambda = 1$.

Mixed strategy Nash equilibrium described in proposition (3) has desirable properties which explain existing variations in tax rates on mobile capital, even among countries which have not yet committed to non-preferential taxation strategies. When cost of capital relocation is sufficiently high both countries set tax rate equal to 1 with strictly positive probability. Also as long as cost of capital relocation is strictly positive, competing countries earn strictly positive tax revenue from mobile capital base as well. Tax competition has reduced tax rate on mobile capital to considerable extent but even today most of countries do impose strictly positive tax rate on highly mobile capital. Perfect capital mobility emerge as limiting case when cost of capital relocation is zero. This leads to Bertrand type outcome where mobile capital is not taxed.

**Proposition (4):** Under preferential regime country’s tax revenue decreases monotonically as $F$ decreases or $\lambda$ increases.

Proposition (4) is similar to proposition (2). If $F < 1/2$ pure strategy Nash equilibrium does not exist. Countries set low tax rate on mobile capital to attract mobile capital from competing country. As $F$ decreases, it becomes easier for capital owners to invest in country with lower tax rate. Increase in $\lambda$ affect the tax revenues of a country directly. Countries compete for bigger fraction of total capital. Because total capital is held constant, this results in lower tax revenues from immobile capital base. Note that under non-preferential regime apart from direct effect emphasized here; $\lambda$ also affect tax revenues indirectly through increase in competition.

### 5 Comparison

In this section we compare the tax revenues generated in equilibrium under preferential regime and non-preferential regime. Head-to-head competition for mobile capital lowers tax revenue from mobile capital under preferential regime compared to non-preferential regime. This is easy to see when $1/2 < F \leq (1 + \frac{1}{\lambda})^{-1}$. Under non-preferential regime pure strategy Nash equilibrium exists, and countries earn tax revenue equal to $\lambda$ from taxes on mobile capital. When governments follow preferential regime, pure strategy Nash equilibrium does not exist. In mixed strategy Nash equilibrium both countries earn $\lambda(1 - \Phi)$, which is strictly less than $\lambda$. For any given $\lambda$, this is true for all values of $F$. This is the basis of arguments against preferential regime. But under preferential regime countries are able to earn higher tax revenue from immobile capital base. Which effect dominates depends on $F$ and $\lambda$. Below in proposition (5), we describe the outcome formally. This is the main result of the paper.

**Proposition (5):** If $F = 0$ then preferential regime as well as non-preferential regime generates equal tax revenues. When $F^1 \leq F \leq F^2$ countries earn higher tax revenues under preferential
regime. If \( F > F^2 \) countries earn higher tax revenues under non-preferential regime. Where,

\[
F^1 = (1 - \lambda) \left[ \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right]^{-1}
\]

and

\[
F^2 = (1 - \lambda) \left[ \left( \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right) - \lambda \left( 1 + \sqrt{2} \right) \right]^{-1}.
\]

First part of proposition (5) confirms the proposition (1) in Wilson (2005). If one tax base is perfectly mobile and other tax base is perfectly immobile then if competing countries are symmetric, non-preferential regime and preferential regime generates equal tax revenues. Comparison differs when mobile capital is imperfectly mobile. We can see that when \( F \) is relatively large, i.e. \( F > F^2 \), countries earn more under non-preferential regime. Opposite is true when \( F^1 \leq F \leq F^2 \), except at the point of equality \( F = F^2 > 0 \), when countries earn equal tax revenue under two regimes. We provide following explanation for results in proposition (5). For any given \( \lambda \) when \( F \) is large, the positive effect of higher tax revenue from mobile capital base, as a result of less competition due to immobile capital base under non-preferential regime dominates. Opposite is true when \( F \) is relatively small. Competing countries have to set low tax rates on mobile and immobile capital bases due to competition for mobile capital base. Loss of tax revenue from low tax rate on immobile capital is more if compared to gain in tax revenue from higher tax rate on mobile capital. In other words, if under preferential regime difference in tax rates between immobile and mobile capital base is large then restriction on preferential regime decreases tax revenues of countries.

It is noteworthy that under non-preferential regime when \( F = F^2 \), countries earn tax revenues equal to \( (1 - \lambda) \) in equilibrium. Even though \( F \) is strictly positive, competition completely erodes tax revenues from mobile capital base. This is not true under preferential regime. Countries expected tax revenues from mobile capital is strictly positive as long as \( F \) is positive.

<table>
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<th>( \lambda )</th>
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Table 1

In table (1) we list \( F^1 \) and \( F^2 \) for different values of \( \lambda \). Preferential regime can generate higher tax revenue compared to non-preferential regime under reasonable circumstances. For example if
70% of capital is mobile then countries will earn higher tax revenue under preferential regime if \( 0.094567 \leq F \leq 0.20237 \) and countries will earn higher tax revenue under non-preferential regime if \( 1 \geq F > 0.20237 \). Note that we compare the outcome of tax competition under preferential and non-preferential regime for \( F > F^1 \).

The result of proposition (5) is explained in figure (4). Function \( F^1 (\lambda) \) explains restriction on \( \lambda \) and \( F \) which insure mixed strategy Nash equilibrium under non-preferential regime. \( F^1 (\lambda) \) shows that for a given \( \lambda \), what is the minimum value of \( F \) such that mixed strategy Nash equilibrium is well defined under non-preferential regime. For a given \( \lambda \) distance between \( F^1 (\lambda) \) and \( F^2 (\lambda) \) shows the range of \( F \) for which preferential regime generates higher tax revenue compared to non-preferential regime. Similarly for \( F \) above \( F^2 (\lambda) \) non-preferential regime generates more tax revenue. From fig (5) we can see that \( F^2 (\lambda) \) is monotonically increasing with \( \lambda \). The reason is that preferential regime dominates non-preferential regime when competition for mobile capital base is intense. Increase in \( \lambda \) intensifies tax competition under non-preferential regime.

6 Conclusion

Literature on tax competition identifies three factors that may explain the gain from non-preferential regime following surprising result in Keen (2001) that restriction of preferential taxation strategy reduces tax revenue of competing country. We identify imperfect capital mobility as another source
of gains from non-preferential regime. In a symmetric game of tax competition we show that non-preferential regime generates higher tax revenue if capital is not highly mobile. On the other hand when capital is highly mobile and mobile capital captures relatively large fraction of total capital preferential regime generates higher tax revenue. When mobile capital is perfectly mobile non-preferential and preferential regime generates equal revenue. When mobile capital is imperfectly mobile our model give more realistic prediction, where, in equilibrium both countries can obtain mobile capital even if their tax rate differ. Also our model describe when non-preferential regime (preferential regime) generates higher tax revenues compared to preferential regime (non-preferential regime) in terms of parameters of the model.

We do not impose restriction to insure pure strategy Nash equilibrium. Mixed strategy Nash equilibrium when competing nations/firms have immobile capital base/captive segment is well understood when a nation/firm can attract investors/consumers through undercutting tax rate/price set by competing nation/firm by small margin. We extend such equilibrium to the case when a nation/firm has to undercut tax rate/price set by competing nation/firm by discrete margin. These results should be useful to a much larger literature in applied game theory and industrial organization.

7 Appendix

Proof of Proposition (1): Strategy pair (1, 1) is pure strategy Nash equilibrium if no country can set tax rate lower then 1 and do strictly better. If a country deviate, he has to lower the tax rate to at least $1 - F$ in order to attract mobile capital from competing country. If a country sets tax rate $1 - F$ it will earn $1 - F + \lambda(1 - F)$. Now for strategy pair (1, 1), to be pure strategy Nash equilibrium it must be true that,

$$1 \geq 1 - F + \lambda(1 - F) \Rightarrow F \left( 1 + \frac{1}{\lambda} \right) \geq 1.$$ 

Thus strategy pair (1, 1) is a Nash equilibrium if $F \geq \left( 1 + \frac{1}{\lambda} \right)^{-1}$. By contradiction, We will show that pure strategy Nash equilibrium does not exist if $F < \left( 1 + \frac{1}{\lambda} \right)^{-1}$. It is clear that (1, 1) is not a pure strategy Nash equilibrium if $F < \left( 1 + \frac{1}{\lambda} \right)^{-1}$. Suppose strategy pair $(\tau, \tau)$ is pure strategy Nash equilibrium where $\tau < 1$. But a country can set tax rate $\tau' = \min \{ 1, \tau + F - \varepsilon \}$ for very small $\varepsilon > 0$, and do strictly better. Thus no symmetric pure strategy Nash equilibrium exists.

Again by contradiction we will show that no asymmetric pure strategy Nash equilibrium exists. Suppose strategy pair $(\tau^1, \tau^2)$ is Nash equilibrium, where $\tau^1$ and $\tau^2$ is tax rate set by country 1 and country 2 respectively. Without loss of generality suppose $\tau^1 < \tau^2$. Then it must be true that $\tau^1 = \tau^2 - F$. Because if $\tau^1 < \tau^2 - F$ then country 1 can do better by setting tax rate equal to $\tau^1 = \tau^2 - F$. But strategy pair $(\tau^2 - F, \tau^2)$ is not a Nash equilibrium because country 2 can do better by decreasing the tax rate slightly. Also if $\tau^2 - F < \tau^1 < \tau^2$ then country 1 can certainly do better by increasing its tax rate to $\tau^2$. This contradict the fact that strategy pair $(\tau^1, \tau^2)$ is a pure
strategy Nash equilibrium. Thus we prove that if \( F < (1 + \frac{1}{\lambda})^{-1} \), pure strategy Nash equilibrium does not exist.

Below we state complete description of part I-III of proposition (1). Each part is followed by its proof.

**I:** If \( \left(1 + \frac{\lambda}{\Psi} + \sqrt{1 + \frac{1}{\lambda}}\right)^{-1} < F < (1 + \frac{1}{\lambda})^{-1} \), then in mixed strategy Nash equilibrium both countries earn tax revenue equal to \( \Omega^N = 1 - \Psi \). Where \( \Psi < F \), is given by eq (4). In mixed strategy Nash equilibrium both countries randomize over the interval \( \{(1 - \Psi - F, 1 - F), (1 - \Psi, 1)\} \). There is probability mass of \( m^p \) at 1 where,

\[
m^p = \frac{\Omega^N - (1 - F)}{\lambda(1 - F)} > 0. \quad (A1)
\]

Equilibrium strategy of countries, i.e. distribution of tax over the support is,

\[
G(\tau) = 1 - \frac{\Omega^N - (1 - \lambda)(\tau + F)}{\lambda(\tau + F)} \text{ for } \tau \in (1 - \Psi - F, 1 - F) \quad (A2)
\]

\[
G(\tau) = 1 - \frac{\Omega^N - (\tau - F)}{\lambda(\tau - F)} \text{ for } \tau \in (1 - \Psi, 1). \quad (A3)
\]

**Proof:** Because equilibrium is symmetric we have not used superscript to differentiate two countries. First of all we need to show that,

\[
\lim_{\varepsilon \to 0} G(1 - F - \varepsilon) = \lim_{\varepsilon \to 0} G(1 - \Psi + \varepsilon). \quad (A4)
\]

Suppose (A4) holds if countries earn tax revenue in mixed strategy Nash equilibrium is \( \Omega \). Then from (A2) and (A3) we have,

\[
1 - \frac{\Omega - (1 - \lambda)((1 - F) + F)}{\lambda((1 - F) + F)} = 1 - \frac{\Omega - ((1 - \Psi) - F)}{\lambda((1 - \Psi) - F)}. \quad (A5)
\]

Solving for \( \Omega \) we get, \( \Omega = \Omega^N \). Thus (A4) holds.

Distribution over the support in the interval \((1 - \Psi, 1)\) is given by (A3). Hence,

\[
G(1) = 1 - \frac{\Omega^N - (1 - F)}{\lambda(1 - F)}
\]

\[
\Rightarrow m^p \equiv 1 - G(1) = \frac{\Omega^N - (1 - F)}{\lambda(1 - F)} > 0 \quad (A6)
\]

(A6) confirms that there is probability mass of \( \frac{\Omega^N - (1 - F)}{\lambda(1 - F)} \) at 1 in equilibrium. Remaining part of the proof we will state in two steps. In step one, we will show that countries earn equal tax revenue everywhere on the support. In step two, we will show that countries can not deviate unilaterally and can do strictly better given that other country sticks to proposed equilibrium strategy.
Step One – If a country sets tax rate equal to 1 with certainty then its expected tax revenue is,

\[ \Omega = (1 - \lambda) 1 + \lambda \left[ 1 - G (1 - F) \right] \quad (A7) \]

\[ \Omega = (1 - \lambda) 1 + \lambda \frac{\Omega^N - (1 - \lambda) (1 - F + F)}{\lambda ((1 - F) + F)} = \Omega^N. \quad (A8) \]

Similarly if a country sets tax rate equal to 1 – F with certainty, its tax revenue in equilibrium is,

\[ \Omega = 1 - F + \lambda m^p (1 - F). \quad (A9) \]

Note that if a country sets tax rate equal to 1 – F, it will retain all domestic capital and it will attract mobile capital from competing country with probability \( m^p \). Using (A1) and (A9) we get, \( \Omega = \Omega^N \).

Similarly if a country sets tax rate \( \tau \) with certainty, s.t. \( \tau \in (1 - \Psi - F, 1 - F) \), then its expected tax revenue is,

\[ \Omega = \tau + \lambda \tau \left[ 1 - G (\tau + F) \right] \quad (A10) \]

Note that \( \tau \in (1 - \Psi - F, 1 - F) \Rightarrow \tau + F \in (1 - \Psi, 1) \). Using (A3) and (A10) we get \( \Omega = \Omega^N \), which is expected tax revenue a country gets in proposed mixed strategy Nash equilibrium. Similarly if a country sets tax rate \( \tau \in (1 - \Psi, 1) \) with certainty its expected tax revenue is,

\[ \Omega = (1 - \lambda) \tau + \lambda \tau \left[ 1 - G (\tau - F) \right]. \quad (A11) \]

\( G (\tau - F) \) is given by (A3) Since \( \tau \in (1 - \Psi, 1) \Rightarrow \tau - F \in (1 - \Psi - F, 1 - F) \). Thus using (A11) and (A2) we get \( \Omega = \Omega^N \), which is expected tax revenue a country gets in mixed strategy Nash equilibrium. This proves that expected tax revenue of competing countries is equal everywhere on the support.

Step Two - Now we will show that a country can not do strictly better by unilateral deviation given other country sticks to proposed equilibrium strategy. Note that \( \text{supremum} \) of the support is 1, which is also the \( \text{maximum} \) tax rate countries can set. Thus we only need to check that a country can not do better if it sets tax rate lower than minimum of the support. Suppose country 1 deviates and set tax rate \( \tau \), s.t. \( 1 - \Psi - F < \tau \leq 1 - 2F \). Its expected tax revenue for such \( \tau \) is,

\[ \Omega \equiv \tau + \lambda \tau \left[ 1 - G (\tau + F) \right]. \quad (A12) \]

Since in equilibrium no country sets tax rate in the range \( (1 - F, 1 - \Psi) \), country 1 does strictly worse if it deviates and sets tax rate \( \tau \), such that \( 1 - \Psi - F < \tau \leq 1 - 2F \). This is because country 1 can raise the tax rate to \( \text{infimum} \) of the support and still collect tax from mobile capital of country 1 and country 2 with same probability.

Now suppose country 1 sets tax rate in the range \( 1 - \Psi - 2F < \tau < 1 - 2F \). Country 1 is able to collect tax from mobile capital of country 2 with probability \( [1 - G (\tau + F)] \), where \( G (\tau + F) \) is given by (A2). Note that, \( \tau \in (1 - \Psi - 2F, 1 - 2F) \Rightarrow \tau + F \in (1 - \Psi - F, 1 - F) \) . From (A2)
and (A12) expected tax revenue of country 1 for $\tau \in (1 - \Psi - 2F, 1 - 2F)$ is,

$$\Omega = \tau + \lambda \frac{\Omega^N - (1 - \lambda)(\tau + F)}{\lambda(\tau + F)},$$

(A13)

$$\Rightarrow \frac{\partial \Omega}{\partial \tau} = \lambda + \Omega^N \left[ \frac{F}{(\tau + F)^2} \right] > 0.$$  

(A14)

From (A14) it is clear that for $\tau \in (1 - \Psi - 2F, 1 - 2F)$ country 1 maximizes its expected tax revenue at $1 - 2F$. But at $1 - 2F$ country 1 earns less compared to expected tax revenue in mixed strategy Nash equilibrium. Hence a country 1 cannot do better by setting tax rate in the range $(1 - \Psi - 2F, 1 - 2F)$.

Now we only need to show that a country cannot do better if it sets tax rate equal to $1 - \Psi - 2F$. If country 1 sets tax rate $1 - \Psi - 2F$, it undercuts tax rate set by country 2 with probability one. Thus to complete the proof we need to show that,

$$(1 + \lambda)(1 - \Psi - 2F) \leq 1 - \Psi$$

(A15)

$$\Rightarrow 1 - \Psi \leq 2F \left( 1 + \frac{1}{\lambda} \right).$$

(A16)

Note that $1 < F \left( 1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right) \Rightarrow 1 - F < F \left( \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right)$. Also we know that $\Psi < F$. Thus sufficient condition for (A16) to hold is,

$$2F \left( 1 + \frac{1}{\lambda} \right) \geq F \left( \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right) + F$$

$$\Rightarrow \left( 1 + \frac{1}{\lambda} \right) \geq \sqrt{1 + \frac{1}{\lambda^2}}, \text{ which is true as } 0 < \lambda < 1.$$

Thus we also prove that country 1 cannot do better by unilateral deviation. By symmetry same is true for country 2 as well. This completes the proof of part (I) of proposition (1).

(II): If $Y \leq F \leq \left( 1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right)^{-1}$, then in symmetric mixed strategy Nash equilibrium both countries earn tax revenue equal to $E \frac{F}{X} \left( 1 + \sqrt{1 + \lambda^2} \right)$. Countries randomize over the interval $\left( \frac{E}{X} \left( 1 + \sqrt{1 + \lambda^2} \right) - F, \frac{E}{X} \left( 1 + \sqrt{1 + \lambda^2} \right) + F \right)$ and there is no probability mass anywhere on the support. Let $\xi \equiv \frac{E}{X} \left( 1 + \sqrt{1 + \lambda^2} \right) - F$ and $\Omega^N \equiv \frac{E}{X} \left( 1 + \sqrt{1 + \lambda^2} \right)$, distribution of tax over the support can be described as,

$$G(\tau) = 1 - \frac{\Omega^N - (1 - \lambda)(\tau + F)}{\lambda(\tau + F)} \text{ for } \tau \in (\xi, \xi + F)$$

(A17)

$$G(\tau) = 1 - \frac{\Omega^N - (\tau - F)}{\lambda(\tau - F)} \text{ for } \tau \in (\xi + F, \xi + 2F).$$

(A18)

Proof - First of all we will show that distribution function is continuous over the support. For
this we need to show that,

\[
\lim_{\varepsilon \to 0} G(\tau + F - \varepsilon) = \lim_{\varepsilon \to 0} G(\tau + F + \varepsilon). \tag{A18}
\]

\[
\lim_{\varepsilon \to 0} G(\tau + F - \varepsilon) \text{ and } \lim_{\varepsilon \to 0} G(\tau + F + \varepsilon) \text{ can be calculated from (A17) and (A18) respectively. Let (A18) holds if } \Omega^N \text{ in (A17) and (A18) is replaced by some arbitrary value } \Omega. \text{ Thus solving (A18) for } \Omega \text{ we get,}
\]

\[
\frac{\Omega - (1 - \lambda)(\tau + F + F)}{\lambda(\tau + F + F)} = \frac{\Omega - (\tau + F - F)}{\lambda(\tau + F - F)} \Rightarrow \Omega = F\left(\frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right). \tag{A19}
\]

This is equal to tax revenue governments earn in mixed strategy Nash equilibrium. This proves that distribution of tax over the support is \textit{continuous}.

Remaining part of proof we state in two steps. In step one we show that a country’s expected tax revenue is equal everywhere on the support and subsequently in step two, we show that a country can not do strictly better by unilateral deviation given that other country sticks to proposed equilibrium strategy.

\textbf{Step One} - Without loss of generality suppose country 1 deviates. If a country 1 sets tax rate \(\tau\) with certainty such that \(\tau \in (\bar{\tau}, \bar{\tau} + F)\) then its expected tax revenue is \(\tau + \lambda \tau \left[1 - G(\tau + F)\right]\). Since \(\tau \in (\bar{\tau}, \bar{\tau} + F)\) implies \(\tau + F \in (\bar{\tau} + F, \bar{\tau} + 2F)\), using (A18) expected tax revenue of country 1 can be written as,

\[
\Omega = \tau + \lambda \tau \left[1 - G(\tau + F)\right] = \tau + \lambda \tau \left[\frac{\Omega^N - \tau}{\lambda \tau}\right] = \Omega^N. \tag{A20}
\]

If country 1 sets tax rate \(\tau\) with certainty such that \(\tau \in (\bar{\tau} + F, \bar{\tau} + 2F)\), its tax revenue will be \((1 - \lambda)\tau + \lambda \tau \left[1 - G(\tau - F)\right]\). Since \(\tau \in (\bar{\tau} + F, \bar{\tau} + 2F)\) implies \(\tau - F \in (\bar{\tau} - F, \bar{\tau} + F)\), using (A17) tax revenue country 1 can be written as,

\[
\Omega \equiv (1 - \lambda)\tau + \lambda \tau \left[\frac{\Omega^N - \tau(1 - \lambda)}{\lambda \tau}\right] = \Omega^N. \tag{A21}
\]

Also the distribution of tax over the support used in equilibrium is \textit{continuous} and there is no probability mass anywhere on the support. Thus from (A20) and (A21) it is clear that country’s expected tax revenue is equal everywhere on the support.

\textbf{Step Two} - Now we show that a country can not do strictly better by setting tax rate outside the support. Without loss of generality suppose country 1 deviates. If country 1 sets tax rate \(\tau \in (\bar{\tau} + 2F, \bar{\tau} + 3F)\), its expected tax revenue will be \((1 - \lambda)\tau + \lambda \tau \left[1 - G(\tau - F)\right]\). Because
If country 1 sets tax rate higher than \( \tau + 3F \), country loses its mobile capital to country 2 with probability one. Thus maximum tax revenue country 1 can earn by setting tax rate higher than \( \tau + 3F \) is \((1 - \lambda)\). This is lower than expected tax revenue country 1 gets in mixed strategy Nash equilibrium. From (A22) it is clear that in the range \((\tau + 2F, \tau + 3F)\) country 1 maximizes its expected tax revenue at infimum of the set. Noting that distribution of tax over the support used by a country in equilibrium is continuous, it is clear that country 1 can not do better by setting tax rate above the proposed support for mixed strategy Nash equilibrium.

Now we show that country 1 can not do better by setting tax rate lower than \( \tau \). If country 1 sets tax rate in the range \((\tau - F, \tau)\), its expected tax revenue is \(\tau + \lambda\tau \left[1 - G(\tau + F)\right]\). Because \((\tau + F) \in (\tau, \tau + F)\), using (A17) expected tax revenue of country 1 can be written as,

\[
\Omega = \tau + \lambda\tau \left[\frac{\Omega^N - (1 - \lambda)(\tau + 2F)}{\lambda(\tau + 2F)}\right] = \lambda\tau + \frac{\Omega^N}{\tau + 2F}.
\]

\[
\Rightarrow \frac{\partial\Omega}{\partial\tau} = \lambda + \frac{2\Omega^N}{(\tau + 2F)^2} > 0.
\]

From (A23) it is clear that country 1 can not do better by setting tax rate lower than infimum of the support. This completes the proof of part (II) of proposition (1).

(III): If \( F = 0 \) then pure strategy Nash equilibrium does not exist. In symmetric mixed strategy Nash equilibrium countries expected tax revenue is \((1 - \lambda)\). In equilibrium countries randomize over the interval \(\left(\left(\frac{1 - \lambda}{1 + \lambda}\right), 1\right)\). Distribution of tax rate over the support used by countries in equilibrium is,

\[
G(\tau) = 1 - \frac{(1 - \lambda)(1 - \tau)}{\lambda\tau} \text{ for } \tau \in \left(\left(\frac{1 - \lambda}{1 + \lambda}\right), 1\right)
\]

Distribution function is continuous and there is no probability mass over the support.

Proof: First of all we will show that country’s expected tax revenue is equal everywhere on the support. If country 1 sets tax rate \( \tau \in \left(\left(\frac{1 - \lambda}{1 + \lambda}\right), 1\right) \), its expected tax revenue can be written as,

\[
\Omega = (1 - \lambda)\tau + \lambda\tau \left[1 - G(\tau)\right]
\]

\[
= (1 - \lambda)\tau + \lambda\tau \left(\frac{(1 - \lambda)(1 - \tau)}{\lambda\tau}\right) = (1 - \lambda)
\]

From (A24) and the fact that the support of mixed strategy Nash equilibrium is convex and probability distribution used in equilibrium is continuous, it is clear that country 1’s expected tax revenue is equal to \((1 - \tau)\) for any tax rate over the support. Also note that if country 1 sets tax
rate equal to $\left(\frac{1-\lambda}{1+\lambda}\right)$, country 1 undercuts tax rate set by country 2 in equilibrium with probability one. Thus country 1 can not do better by setting tax rate outside the support. From symmetry same will be true for country 2 as well. This completes the proof of part (III) of proposition (1).

QED.

Proof of Proposition (2): As long as $F \geq (1 + \frac{1}{\lambda})^{-1}$ holds, any change in $F$ or $\lambda$ will have no effect on expected tax revenues countries earn in equilibrium. The reason is that pure strategy Nash equilibrium exists and both countries set tax rate equal to 1.

If $\left(1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right)^{-1} < F < (1 + \frac{1}{\lambda})^{-1}$ then expected revenue in equilibrium is $E(R) \equiv 1 - \Psi$, where $\Psi$ is given by (4). Differentiating $E(R)$ with respect to $F$ and $\lambda$ we have

$$\frac{\partial E(R)}{\partial F} = \frac{1}{2} + \frac{1}{2} \frac{F + (1 + \lambda)}{\sqrt{F^2 + 2F(1-\lambda) + (1-\lambda)^2}} > 0 \quad (A25)$$

$$\frac{\partial E(R)}{\partial \lambda} = \frac{F - (1-\lambda)}{2\sqrt{F^2 + 2F(1+\lambda) + (1-\lambda)^2}} - \frac{1}{2} < 0. \quad (A26)$$

In (A25) both numerator and denominator are greater than zero, as $0 < \lambda < 1$. This implies that right hand side of (A25) is strictly greater then zero.

Note that in (A26) if $F - (1-\lambda) \leq 0$, because second term of right hand side is negative, whole term will be negative as well. But if $F - (1-\lambda) > 0$ right hand side is negative if first term is less then $\frac{1}{2}$. This is true if,

$$\sqrt{F^2 + 2F(1+\lambda) + (1-\lambda)^2} > F - (1-\lambda)$$

iff, $F^2 + 2F(1+\lambda) + (1-\lambda)^2 > F^2 + (1-\lambda)^2 - 2F(1-\lambda)$

$$\Rightarrow 4F > 0$$ which is true for $F > 0$.

If $\Upsilon \leq F \leq \left(1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}}\right)^{-1}$, expected tax revenue in equilibrium is $E(R) \equiv \frac{F}{\lambda} \left(1 + \sqrt{1 + \lambda^2}\right)$. Differentiating $E(R)$ with respect to $F$ and $\lambda$ respectively we get,

$$\frac{\partial E(R)}{\partial F} = \frac{1}{\lambda} + \sqrt{\frac{1}{\lambda^2}} > 0 \quad (A27)$$

$$\frac{\partial E(R)}{\partial \lambda} = -F \left[\frac{1}{\lambda^2} + \frac{2}{\lambda^3 + \sqrt{1 + \frac{1}{\lambda^2}}}\right] < 0. \quad (A28)$$

This completes the proof. Q.E.D.

Proof of Proposition (3): Under preferential regime countries set tax rate on mobile capital and immobile capital independently. Countries maximize tax revenue from immobile capital by setting highest possible tax rate (equal to 1). Since fraction $(1 - \lambda)$ of capital is immobile, countries earn tax revenue of $(1 - \lambda)$.

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Now there is head-to-head competition for mobile capital. Nature of equilibrium is same if one substitute $\lambda = 1$ in proposition (1). Since fraction $\lambda$ of total capital is mobile, countries earn tax revenue $\lambda \cdot R$, where $R$ is tax revenue countries earn in equilibrium in proposition (1) if $\lambda = 1$. Note that if $\lambda = 1$ then $\frac{F}{(1-\lambda)} \left( \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right) = \infty$. Thus we do not need the restriction imposed on $F$ under non-preferential regime to find mixed strategy Nash equilibrium. Thus proof of part (I) and part (II) of proposition (3) are similar respectively to part (I) and part (II) of proposition (1). Proof of part (III) of proposition (3) is trivial. This case is similar to Bertrand price competition. This completes the proof. $\text{Q.E.D.}$.

Proof of Proposition (4): As long as $F \geq \tau/2$, is satisfied any change in $F$ or $\lambda$ will not affect the tax revenue. This is because pure strategy Nash equilibrium exists and countries earn tax revenues equal to 1 in equilibrium. If $1/(2 + \sqrt{2}) < F < 1/2$ then expected tax revenue of countries is $E(R) \equiv (1 - \lambda) + \lambda (1 - \Phi)$, where $\Phi$ is given by (7). Differentiating $E(R)$ with respect to $F$ and $\lambda$ respectively we get,

$$\frac{\partial E(R)}{\partial F} = \lambda \left[ \frac{1}{2} + \frac{F + 2}{2\sqrt{F^2 + 4F}} \right] > 0 \quad (A29)$$

$$\frac{\partial E(R)}{\partial \lambda} = \frac{F + \sqrt{F^2 + 4F} - 2}{2} < 0. \quad (A30)$$

If $F \leq 1/(2 + \sqrt{2})$, expected tax revenue of a country in equilibrium is $E(R) \equiv (1 - \lambda) + \lambda F(1 + \sqrt{2})$. Differentiating $E(R)$ with respect to $F$ and $\lambda$ respectively we get,

$$\frac{\partial E(R)}{\partial F} = \lambda \left( 1 + \sqrt{2} \right) > 0 \quad (A31)$$

$$\frac{\partial E(R)}{\partial \lambda} = F \left( 1 + \sqrt{2} \right) - 1 < 0. \quad (A32)$$

This completes the proof. $\text{Q.E.D.}$.

Following claims below will help us prove proposition (5).

Claim (1): When $1/2 \leq F$ countries earn equal tax revenue under two regimes. When $(1 + \frac{1}{\lambda})^{-1} \leq F < 1/2$ non-preferential regime, generate higher tax revenues.

Proof - Note that under non-preferential regime pure strategy Nash equilibrium exists as long as $F \geq (1 + \frac{1}{\lambda})^{-1}$, and countries earn tax revenue equal to 1 in equilibrium. Under preferential regime pure strategy Nash equilibrium exists only if $F \geq 1/2$. Note that $(1 + \frac{1}{\lambda}) > 2$ as $0 < \lambda < 1$. Hence when $F \geq 1/2$, pure strategy Nash equilibrium exists under both regimes and countries earn tax revenue equal to 1. But when $(1 + \frac{1}{\lambda})^{-1} \leq F < 1/2$, pure strategy Nash equilibrium does not exist under preferential regime and countries earn tax revenue less then $\lambda$ from mobile capital base, even though they continue to earn $(1 - \lambda)$ from tax immobile capital base, hence total tax revenues fall below 1 under preferential regime. This proves claim (1). $\text{Q.E.D.}$.

Claim (2): If $1/(2 + \sqrt{2}) \leq F < (1 + \frac{1}{\lambda})^{-1}$ countries earn higher tax revenues under non-preferential regime.
Proof - First of all note that,

$$\left(1 + \frac{1}{\lambda}\right)^{-1} > 1 / \left(2 + \sqrt{2}\right) \Rightarrow \lambda > \sqrt{2} - 1.$$  

When $\lambda > \sqrt{2} - 1$ for $1 / (2 + \sqrt{2}) \leq F < (1 + \frac{1}{\lambda})^{-1}$, under non-preferential regime, countries earn tax revenues equal to,

$$\frac{1}{2} (1 - \lambda) + \frac{F}{2} + \frac{1}{2} \sqrt{F^2 + 2F (1 + \lambda) + (1 - \lambda)^2}$$

and under preferential regime countries earn tax revenue equal to

$$(1 - \lambda) + \lambda \left(\frac{F}{2} + \frac{1}{2} \sqrt{F^2 + 4F}\right).$$

We need to show that,

$$\Lambda \equiv \frac{1}{2} (1 - \lambda) + \frac{F}{2} + \frac{1}{2} \sqrt{F^2 + 2F (1 + \lambda) + (1 - \lambda)^2}$$

$$- (1 - \lambda) 1 - \lambda \left(\frac{F}{2} + \frac{1}{2} \sqrt{F^2 + 4F}\right) \geq 0,$$

or equivalently,

$$\frac{\Lambda}{\Gamma} \equiv -\frac{F}{2} (1 - \lambda) + \frac{1}{2} + \frac{1}{2} \sqrt{1 + 2 \Gamma (1 + \lambda) + (\Gamma)^2 (1 - \lambda)^2}$$

$$- \lambda \left(\frac{1}{2} + \frac{1}{2} \sqrt{1 + 4\Gamma}\right) \geq 0$$

where $\Lambda$ denotes difference between tax revenues under non-preferential regime and preferential regime and $\Gamma = \frac{1}{F}$. Also note that,

$$F \left(1 + \frac{1}{\lambda}\right) < 1 \leq F \left(2 + \sqrt{2}\right) \Leftrightarrow \left(1 + \frac{1}{\lambda}\right) < \Gamma \leq \left(2 + \sqrt{2}\right).$$

We solve equation $\frac{\Lambda}{\Gamma} = 0$ for $\Gamma$ when $0 < \lambda < 1$. We denote two solutions of this equation as $\Gamma^1$ and $\Gamma^2$ where,

$$\Gamma^1 = \left\{2\lambda \left(2\lambda + \frac{1}{\lambda} \left(1 + \lambda + \sqrt{1 + 4\lambda^2}\right) + \sqrt{1 + 4\lambda^2}\right)\right\}$$

and,

$$\Gamma^2 = \left\{\frac{1}{2\lambda} \left(2\lambda + \frac{1}{\lambda} \left(1 + \lambda - \sqrt{1 + 4\lambda^2}\right) + \sqrt{1 + 4\lambda^2}\right)\right\}.$$
Note that,

\[ \Gamma^1 = 2 + 4\lambda^2 + 2\lambda + 2\sqrt{1 + 4\lambda^2} + 4\lambda\sqrt{1 + 4\lambda^2} \]
\[ \Rightarrow \Gamma^1 > 4 \text{ as } 0 < \lambda < 1. \]

Similarly,

\[ \Gamma^2 = 1 + \frac{1}{2\lambda^2} + \frac{1}{2\lambda} - \sqrt{1 + \frac{1}{4\lambda^2}} - \frac{1}{\lambda} \sqrt{1 + \frac{1}{4\lambda^2}}, \]
\[ \Rightarrow \Gamma^2 < 1 + \frac{1}{2\lambda^2} + \frac{1}{2\lambda} - \frac{1}{2\lambda^2} \text{ as } \sqrt{1 + \frac{1}{4\lambda^2}} > \frac{1}{2\lambda}, \]
\[ \Rightarrow \Gamma^2 < 1 \Rightarrow \Gamma^2 < \left( 1 + \frac{1}{\lambda} \right). \]

Thus there is no \( \Gamma \) st. \( (1 + \frac{1}{\lambda}) < \Gamma \leq (2 + \sqrt{2}) \), which is solution to the equation \( \frac{A}{F} = 0 \). Also it is known from claim (1), that for \( \Gamma = (1 + \frac{1}{\lambda}) \), countries earn higher tax revenue under non-preferential regime. Because tax revenue in equilibrium is continuous, it must be the case that countries earn higher tax revenue for \( \Gamma = (1 + \frac{1}{\lambda}) + \epsilon \), for some small \( \epsilon > 0 \). Thus in the interval \( (1 + \frac{1}{\lambda}) < \Gamma \leq (2 + \sqrt{2}) \), we must have \( \Lambda > 0 \) at the infimum of the set and \( \Lambda \neq 0 \) for any \( \Gamma \) such that \( (1 + \frac{1}{\lambda}) < \Gamma \leq (2 + \sqrt{2}) \). This proves claim (2).

**Claim (3):** When \( (1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}})^{-1} \leq F < 1/(2 + \sqrt{2}) \) countries earn higher tax revenue under non-preferential regime compared to preferential regime.

**Proof** - First of all note that,

\[ (2 + \sqrt{2}) < \left( 1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right) \text{ for } \forall \lambda \text{ s.t. } 0 < \lambda < 1. \]

If \( \lambda \leq \sqrt{2} - 1 \) then if \( (1 + \frac{1}{\lambda})^{-1} \leq F < 1/(2 + \sqrt{2}) \), pure strategy Nash equilibrium exist under non-preferential regime. Because in pure strategy Nash equilibrium a country earns maximum possible tax revenues, it is trivial that under non-preferential regime countries earn higher tax revenues.

For such a range of \( F \), tax revenue under non-preferential regime \( (R_{NP}) \) and preferential regime \( (R^p) \) is given as,

\[ R_{NP} = \frac{1}{2} (1 - \lambda) + \frac{F}{2} + \frac{1}{2} \sqrt{F^2 + 2F1(1 + \lambda) + (1)^2 (1 - \lambda)^2}, \]
\[ R^p = 1 (1 - \lambda) + F \left( 1 + \sqrt{2} \right) \lambda. \]
We need to show that,

\[ \Lambda = R^{NP} - R^p > 0 \text{ for } F \left( 1 + \frac{1}{\lambda} \right) < 1 \leq F \left( \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right) + F \text{ if } \lambda \leq \sqrt{2} - 1, \]

or, \[ \Lambda = R^{NP} - R^p > 0 \text{ for } F \left( 2 + \sqrt{2} \right) < 1 \leq F \left( \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right) + F \text{ if } \lambda > \sqrt{2} - 1. \]

Now \( \Lambda > 0 \) if,

\[ \sqrt{1 + 2\Gamma (1 + \lambda) + \Gamma^2 (1 - \lambda)^2} \geq \Gamma (1 - \lambda) + 2 \left( 1 + \sqrt{2} \right) \lambda - 1 > 0 \text{ where } \Gamma = \frac{1}{F} \]

\[ \Rightarrow \Gamma > \frac{\lambda (1 + \sqrt{2}) [\lambda (1 + \sqrt{2}) - 1]}{1 - \lambda (1 - \lambda) (1 + \sqrt{2})} \]

Note that \( 1 - \lambda (1 - \lambda) (1 + \sqrt{2}) > 0 \) for all \( \lambda \) s.t., \( 0 < \lambda < 1 \). Thus when \( \lambda \leq \sqrt{2} - 1 \), numerator is less than zero and denominator is greater than zero. Thus condition holds trivially. Now for \( \lambda > \sqrt{2} - 1 \), both numerator and denominator are positive. Thus to prove the claim we only need to show that,

\[ \frac{\lambda (1 + \sqrt{2}) [\lambda (1 + \sqrt{2}) - 1]}{1 - \lambda (1 - \lambda) (1 + \sqrt{2})} \leq 2 + \sqrt{2} \]

\[ \Rightarrow \lambda - \lambda^2 - \frac{2 + \sqrt{2}}{1 + \sqrt{2}} < 0. \]

Now let \( \Phi = \lambda - \lambda^2 - \frac{2 + \sqrt{2}}{1 + \sqrt{2}} \). We can see that \( \Phi (\lambda) \) is concave function of \( \lambda \).

\[ \max \Phi (\lambda) = \frac{1}{2} - \frac{1}{4} - \frac{2 + \sqrt{2}}{(1 + \sqrt{2})^2} < 0. \]

Thus we have \( \Lambda > 0 \). This proves the claim (3).

Proof of Proposition (5). From claim 1-3 it is clear that as long as \( \left( 1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right)^{-1} < F \) countries earn higher tax revenue under non-preferential regime compared to preferential regime. The relation is strict if \( \left( 1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right)^{-1} < F < 1/2 \). Now to prove the proposition (3), we will compare the tax revenue for countries when \( \left( 1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right)^{-1} \geq F \).

When \( \left( 1 + \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right)^{-1} \geq F \geq (1 - \lambda) \left( \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right)^{-1} \) then under non-preferential regime countries earn tax revenue equal to \( F \left( \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right) \) and under preferential regime countries earn tax revenue equal to \( (1 - \lambda) 1 + F (1 + \sqrt{2}) \lambda \). As before let \( \Lambda \) denotes the difference
between tax revenues under non-preferential regime and preferential regime. Thus we have,

\[ \Lambda = F \left( \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right) - (1 - \lambda) 1 - F \left( 1 + \sqrt{2} \right) \lambda \geq 0 \]

iff,

\[ 1 \leq \frac{1}{(1 - \lambda)} \left[ F \left( \frac{1}{\lambda} + \sqrt{1 + \frac{1}{\lambda^2}} \right) - \lambda F \left( 1 + \sqrt{2} \right) \right]. \]  
(A33)

Rearranging (A33) we get, \( \Lambda < 0 \) if \( F^1 \leq F \leq F^2 \) and \( \Lambda > 0 \) otherwise, where \( F^1 \) and \( F^2 \) are given by (8) and (9) respectively. Thus when \( F^1 \leq F \leq F^2 \) countries earn higher tax revenue under preferential regime and if \( F > F^2 \), countries earn higher tax revenue under non-preferential regime. This completes the proof.

\[ Q.E.D. \]
References


[34] Wilson, J.D. (1999), Theories of Tax Competition, National Tax Journal 52, 269-304.

