# Financial volatility and its economic effects

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#### Abstract

I empirically investigate whether increases in the uncertainty about the performance of financial firms worsens their agency and informational problems, reducing their ability to intermediate funds, thus, protracting economic activity. In order to do so, I build a dynamic stochastic general equilibrium model, deriving from it sign restrictions identifying exogenous disturbances to financial uncertainty, and then use these restrictions in a medium sized vector-autoregression with several aggregate variables. The results show that these shocks do not account for large shares in the variations of key macroeconomic variables (e.g. GDP, investment and hours worked). However, when they occur, they result in significant and persistent economic effects, such as drop of 1.5% in investment lasting for more than 2 years. Furthermore, when focusing on specific periods of time, they explain 35% to 40% of the decrease in economic activity during the Great Recession.

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## 1 Introduction

Is the financial sector an important source of macroeconomic fluctuations or does it only propagate events from elsewhere in the economy? During the Great Recession, we observed a severe drop in equity from banks, a collapse of interbank markets and massive failures of financial institutions, among which Lehman Brothers is the most famous example. A possible explanation is that these negative events in the financial system were simply a consequence of the already declining aggregate output. However, an alternate possibility is that problems within the financial sector may have further pushed the economy into an even deeper recession.

In this paper, I explore the latter by empirically investigating whether increases in the uncertainty about the performance of financial firms worsens these firms' agency and informational problems, reducing their ability to intermediate funds, thus, protracting economic activity. In order to do so, I build a dynamic stochastic general equilibrium (DSGE) model, deriving from it sign restrictions identifying exogenous disturbances to financial uncertainty, and then use these restrictions in a medium sized vector-autoregression (VAR) with several aggregate variables. The results show that these shocks do not account for large shares in the variations of key macroeconomic variables (e.g. GDP, investment and hours worked). However, when they occur, they result in significant and persistent economic effects, such as drop of 1.5% in investment lasting for more than 2 years. Furthermore, when focusing on specific periods of time, they explain 35% to 40% of the decrease in economic activity during the Great Recession.

In order to reach these conclusions, the first challenge is to choose an empirical measure of the uncertainty about the performance of firms that is both micro-founded by a theoretical framework and informative about uncertainty shocks faced by these firms. I show that the simple standard deviation of stock market returns across firms in each time period (e.g. month or quarter)—throughout this paper denoted by *volatility*—satisfies these conditions.

A first look at the data shows us that uncertain times for the financial sector are associated with poor aggregate economic performance. Higher *financial volatility*—i.e. the standard deviation of stock returns among financial firms—is associated with lower credit, higher funding costs for both financial and non-financial firms, and a greater number of failing financial firms. Additionally, financial volatility is counter-cyclical, with sizable negative correlations with GDP, investment and hours worked. However, in order to reach causal conclusions, the empirical approach needs to disentangle the increases in financial volatility originated in the financial sector from those that are just a reaction to events from elsewhere in economy. I then build a DSGE model providing the economic foundations for extracting causal inferences from the data. The key feature of the model is that it allows for agency problems, such as the one originally proposed by Townsend (1979) and further explored by Bernanke et al (1999), to financial firms. In order to intermediate funds, these firms need to finance themselves with loan contracts under idiosyncratic risk. The idea is to proxy for the fact that, in reality, different financial firms use different credit intermediation practices (e.g. statistical models, credit scores and personal relationships with clients) achieving varying degrees of diversification on their assets. The extent to which these firms may experience different rates of return on their *assets* is denoted as *financial volatility shock*. This is meant to capture exogenous changes in the dispersion of banking practices and disruptions in financial markets precluding financial firms to implement their hedging strategies, thus exposing them to a greater risk of bankrupcy. I also allow non-financial firms to face agency problems in an analogous manner, denoting *non-financial volatility shock* as the dispersion of asset returns on non-financial activities.

The model contributes twofold to the empirical analysis. First, it allows me to show that there is a counterpart in the model for the empirical standard deviation of stock returns of both financial and non-financial firms, thus micro-founding the choice for the empirical uncertainty measure. Second, by studying the signs of the reactions of model variables to several strutural shocks, I am able to distinguish a financial volatility shock from a non-financial one, as well as from many other shocks used in the literature. This is important because it provides me with the choice of variables to be used in the VAR and with an empirical strategy able to identify the volatility shocks in the data.

Finally, I estimate a reduced form VAR with several aggregate macro (e.g. GDP, investment, credit) and financial (e.g. financial and non-financial volatilities) variables and impose sign restrictions in order to identify the volatility shocks. The advantages of this approach are twofold. First, it does not require the identifying restrictions to be strong enough to fully identify the shock of interest; I am allowed to focus on a set of qualitative implications for the data, possibly only achieving a partial identification of the shock. Second, there is no need to make assumptions about shocks other than the one focused on this paper. As a matter of comparison, when estimating a DSGE model, the researcher needs to specify a number of shocks at least as large as the number of observed variables used in the estimation, even if the focus of the study is only one of these shocks.

Equipped with the framework above, I ask the following questions:

(i) Does an exogenous financial volatility shock imply significant effects on the economy? If so, are those effects consistent with those predicted by the model?

The results are quantitatively very similar to the model's predictions. We observe a rise in the standard deviation of returns of financial and non-financial firms. There is a persistent decline in the amount of credit available to non-financial firms, reaching its trough after 2 years at a level 0.7% lower than the initial one. The interest rate on loans rises, with the one charged to non-financial firms experiencing the highest and most persistent increase. The production side of the economy follows with GDP, investment and hours worked reaching troughs after a year at levels 0.4%, 1.5% and 0.5% lower than before. These variables only return to their initial level after 11, 9 and 12 quarters, respectively. Finally, there is an aggressive response of the Fed Funds, following a pattern similar to the one previously described.

(ii) To what extent can we account for the variation in key macroeconomic variables (i.e. GDP and investment) using financial volatility shocks?

This shock explains between 20% to 30% of variables measuring economic activity (GDP, investment and hours worked), and between 10% to 30% of several others. However, these results are subject to relatively large probability intervals, with the distance between the 10<sup>th</sup> and 90<sup>th</sup> quantiles ranging between 20% and 50%. Focusing on the lower 10<sup>th</sup> percentile of explanatory power, the numbers range from approximately 5% to 12%. One important conclusion from these results is that a large share of the observed financial volatility is explained by other shocks, meaning that it should not be taken as an approximate proxy for its exogenous component.

(iii) If financial volatility shocks are a relevant source of macroeconomic fluctuations, can we pinpoint specific instances when they occurred?

In order to pin down a specific time period when a financial volatility shock was relevant, I implement a historical decomposition. I estimate how the economy would have behaved if it had been exclusively driven by financial volatility shocks during the last three downturns: early 1990's, early 2000's and the Great Recession. The latter leads to the strongest results with the financial volatility shock explaining 35-40% of the observed trajectory of variables measuring economic activity (GDP, investment and hours worked), 32% of consumption, 30% of credit, 20-25% of the funding costs for financial and non-financial firms and 36% of the financial volatility.

#### (iv) What is the role of non-financial volatility shocks?

They singlehandedly do not seem to explain a large variation of the data. However, when I use them to evaluate the *joint* role of volatility shocks for the data, I find that both types of shocks account for 40% of the variation of variable measuring economic activity during the sample. Additionally, these shocks account for approximately 50% of the fall in aggregate output during the Great Recession.

This paper is organized as follows. In Section 1.1, I relate the contributions of this paper within the context of the literature. In Section 2, I present evidence of how financial volatility correlates with key macro and financial variables, while in Section 3, I build a DSGE model guiding me with the empirical exercises in the rest of the paper. In Section 4, I use the theoretical framework of the previous section to show why the standard deviation of stock market returns is informative about the volatility shocks studied here. Additionally, I discuss the sign restrictions predicted by the model that identify the volatility shocks. Section 5 briefly addresses the methodology of using reduced form VARs with sign restrictions. Section 6 presents the empirical results.

#### **1.1 Related literature**

Among the papers studying the empirical effects of uncertainty shocks in aggregate variables, most of them focus on the non-financial sector. Moreover, there are many different empirical strategies. Some researchers do not use any specific model to guide their analysis and focus on variables proxying the dispersion of beliefs among the agents of the economy (e.g. Popescu and Smets (2010), Bachmann et al (2010) and Alexopoulos and Cohen (2009)). Other papers have models underlying the study, but it is not clear how the specific measure used maps into the proposed theoretical framework (e.g. Bloom (2009)). I follow a third and more recent path of the literature, focusing on a specific empirical measure that has a counterpart in the model underlying the analysis (e.g. Bachmann and Bayer (2009) and Bloom et al (2012)). There are some papers considering the possibility of financial uncertainty shocks, for instance Nowbilski (2012), Hirakata et al (2009), Hirakata et al (2011), Christiano and Ikeda (2011) and Zeng (2011). However, none of them implement empirical investigations, with the exception of Hirakata et al (2011). The latter estimate a DSGE model with both financial and non-financial volatility shocks but they do not find evidence that financial volatility shocks are empirically relevant. Besides the empirical approach, the key difference between such study and this paper is that they do not use time series of standard deviation of stock returns.

This paper also relates to large literature on financial frictions applied to financial firms. Among them are: Holmstrom and Tirole (1997), Kiyotaki and Moore (2008), Gertler and Karadi (2009), Gertler and Kyotaki (2011). Even though Bernanke et al (1999) provide a framework of financial frictions to non-financial firms, many papers, including this one, have also extended such approach for financial firms. Thus, one consequence of the empirical results from this study is that it provides evidence supporting Bernanke's et al (1999) framework within the financial sector.

Finally, there are also several papers documenting micro evidence of a credit supply shock during the Great Recession, such as Santos (2011), Adrian et al (2013), Chodorow-Reich (2012) and Ivashina and Scharfstein (2010). This paper supports such evidence using aggregate data and a strutural approach.

## 2 Financial volatility

Before diving into the structural analysis of the data, it is useful to check how the measure of uncertainty about the performance of *financial* firms used in this paper correlates with key macro and financial variables along the business cycle. This is important because it give us an indication about how financial uncertainty shocks might affect the rest of the economy. On the other hand, since the focus of the related literature has been on the non-financial sector, it is also important to contrast these correlations with those calculated using an uncertainty measure about the performance of *nonfinancial* firms. This analysis shows that uncertain times for financial firms are associated with lower credit growth, higher funding spreads, lower economic growth and greater number of failing financial firms, suggesting an effect via a negative credit supply shock to the economy. The correlations for the non-financial sector are similar, but with smaller magnitudes. These results largely anticipate those found with the strutural framework: uncertainty shocks to the financial sector imply in larger and more persistent effects than those to the non-financial one.

Throughout this paper, I use the *un-weighted* standard deviation of stock returns across firms

within each time period (e.g. month, quarter) as the the measure of uncertainty about the performance of firms. The micro-foundation for this specific measure is discussed in Section 4.1. I denote as *financial volatility* the standard deviation of returns of financial firms, while *non-financial volatility* is the equivalent for non-financial firms. Stock returns are taken from the CRSP database from January 1980 to December 2010 and the details about the calculations of these variables are provided in Appendix A.

Table 1 lists the correlations of financial and non-financial volatility with several macro and financial variables. It shows that both volatility measures have moderate to sizable correlations with measures of economic activity (GDP, investment and hours worked) and are associated with worsening conditions on the credit market for non-financial firms: negative correlations with credit and positive with funding costs, here measured by the Baa and commercial paper spreads. However, there are two additional evidences suggesting that the financial measure may contain information specific to the financial sector. First, it has a correlation with funding costs to financial firms (Certificate of Deposits spread and commercial paper) much higher than the one held by the nonfinancial measure. Second, there is a larger increase in failures of financial firms when financial volatility increases than when the non-financial one does. Lastly, a fact worth noting is that the correlations presented in Table 1 are generally higher for the financial measure than for the nonfinancial one.

In order to provide better visualization of the volatility measures used here, Figures 1a and 1b show their time series. Figure 1a also includes the log of real GDP detrended using a Hodrick-Prescott filter for a more precise comparison with the business cycle. By analyzing these figures, we notice that both volatility measures present sizable increases during the last four recessions and that they present a considerable co-movement, with a correlation of 0.62 among them. However, there are also marked differences between these variables. First, while the mean of the financial volatility over the sample (where returns are in quarterly terms) is 17.7%, the equivalent for non-financial firms firms is 28.9%. Second, while the highest peak of the non-financial volatility occurs during the early 2000's recession, the two highest peaks for the financial one occurred during recessions in which the health of the financial system was under heavy debate: the early 1990's recession after the Savings and Loans crisis and the 2008-2009 recession during the Subprime crisis. These remarks suggest that these last two recessions might have different origins, a conjecture confirmed by the strutural analysis done later showing that volatility shocks played a significant role in the Great Recession, while not so much in the others.

Variable	GDP <sup>a</sup>	$Investment^a$	Hours worked <sup>a</sup>	Credit to Certificate		Financial	-	Non-financial	Failure of
				non-financial	of deposit	commercial	Baa	commercial	financial
				$\mathbf{firms}^b$	$\operatorname{spread}^c$	paper $\operatorname{spread}^d$	spread <sup>e</sup>	paper $\operatorname{spread}^d$	${\rm firm}{\rm s}^f$
Financial	-0.33	-0.45	-0.25	-0.32	0.24	0.34	0.40	0.26	0.52
Non-financial	-0.25	-0.40	-0.20	-0.27	-0.08	0.13	0.38	0.16	0.15
Frequency	q	q	q	q	m	m	m	m	у

Table 1: Correlation of macro and financial variables with volatilities

See Appendix A for detailed definition of all variables. The row "Frequency" describes whether the data is quarterly, "q", monthly, "m", or yearly, "y". <sup>a</sup>Regarding GDP, investment and hours worked, I take logs and remove their trends using a Hodrick-Prescott filter with parameter 1600 (standard for quarterly data) before calculating their correlations. I have also used much higher parameter values, proxying for a linear detrending, and while the correlations with the financial volatility are highly insensitive to such increase, those with the non-financial volatility experience a decrease in their magnitudes. <sup>b</sup>As for credit to non-financial firms, I use it in log-growth rates. <sup>c</sup>Certificate of deposits spread is the difference between the rates on 3 month certificate of deposit and the 3 month Treasury bill. <sup>d</sup>Commercial paper spreads for financial and non-financial firms discount the rate of the 3 month Treasury bill. <sup>e</sup>The Baa spread is the difference between Moody's Baa and the 10 year Treasury constant maturity rates. <sup>f</sup>Failure of financial firms is measured by the ratio between total of assets of all institutions Failed/Assisted by the FDIC divided by total assets of financial business from the Flow of Funds. I use the average of the monthly volatility measures over the year to compute the correlations with the measure for failures of financial firms.

## 3 Model

The objective of the model described in this section is to provide the economic foundations for the empirical analysis performed in the rest of the paper. It does so by achieving two goals. First, it precisely defines the volatility shocks studied here and explains how these shocks should translate into relevant economic effects. Second, it delivers the theoretical underpinnings for the identification of these shocks in the data and the selection of variables used in such empirical investigation. In particular, it gives micro-foundation for the choice of the standard deviation of stock returns as the measure of uncertainty about the performance of firms.

The model is an otherwise new Keynesian framework with an imperfect credit market in which both financial and non-financial firms face frictions to finance their activities. These financial frictions are modeled by debt contracts like those explored in Bernanke et al (1999) (henceforth BGG) and originally proposed by Townsend (1979). The reason for introducing both of these frictions is to endow the model with two types of volatility shocks: *financial* and *non-financial*<sup>1</sup>. This allows me

<sup>&</sup>lt;sup>1</sup>There are many other models exploring the contracting framework proposed by Townsend (1979) in a DSGE model where there is time varying dispersion of returns for borrowers. Among these are Christiano et al (2013), Nowobilski (2012), Hirakata et al (2011) and Christiano and Ikeda (2011). Some of these models only introduce credit frictions either on financial firms or on the non-financials ones. Other models make two sided contracts between depositors, banks and

to explore the different reactions of several model variables to these shocks and use these model implications to empirically identify the *financial* shock while controling for effects from the *nonfinancial* one.

These volatility shocks have significant economic effects in this framework because they increase lending risk. The intuition for this comes from the fact that lenders write loan contracts without knowing the rate of return of the investments of each one of their debtors. However, these lenders have information about the mean and dispersion of these returns. Then, they anticipate that the higher the dispersion, the larger the number of borrowers failing to repay their loans. Consequently, in periods of higher volatility of returns, lenders decrease their loan amounts and increase the interest rates charged. In turn, this decreases resources available for investment in stock of capital and the economy experiences a downturn. Notice that these co-movements among model variables are consistent with the empirical ones reported in Table 1.

The overview of the model is as follows<sup>2</sup>. There are five types of agents participating in financial markets: households, mutual funds, financial firms, loan brokers and non-financial firms, here called entrepreneurs. Households choose how much to save, consume and accumulate capital. A large number of mutual funds, then, borrows these savings and competes in the loans market to lend to financial firms. The latter, in turn, invest in loan brokers, who also compete to lend to entrepreneurs. Entrepreneurs use their loans to purchase capital from households, rent it to intermediate firms and re-sell it to households.

The rest of the model has features enhancing its ability to propagate the volatility shocks. Among these features are sticky wages, dynamic adjustment costs in investment and habit formation in consumption. These frictions help the model to generate co-movements among key variables (e.g. hours worked, invesment, consumption) quantitatively similar to those observed in the data. In particular, they help the model deliver impulse response functions very close to those found in the data. On the other hand, only the adjustment cost in investment is necessary to provide the foundations for the empirical identification of the volatility shocks. These additional frictions are described in detail in Appendix D. The rest of this section is divided into two parts. In the first, I analyze the contracts by which mutual funds finance loans to financial firms, while in the second, I describe how loan brokers fund themselves from financial firms and lend to entrepreneurs.

non-financial firms. Here, I introduce frictions both on financial and non-financial while keeping the debt contracts as close as possible to the one originally used by BGG.

 $<sup>^{2}</sup>$ For a graphical representation of the model, see Figure 2.

### 3.1 Contract between mutual funds and financial firms

There is a unit measure of financial firms. At the end of period t, financial firm i gets a loan  $(B_{i,t+1}, Z_{i,t+1}^f)$  from a mutual fund, where  $B_{i,t+1}$  is the loan amount and  $Z_{i,t+1}^f$  is the state-contingent interest rate. With end-of-period-t equity  $N_{i,t+1}^f$  and total assets  $A_{i,t+1} = N_{i,t+1}^f + B_{i,t+1}$ , financial firm i decides its investment option. The first alternative is to invest its assets in loan brokers' funds, which lends to entrepreneurs. In such a case, firm i earns an average return of  $R_{t+1}^f$ . The second alternative is to replicate the business strategy of an entrepreneur and earn a rate of return  $R_{t+1}^e$  by purchasing, renting and re-selling physical capital services. However, I assume that financial firms can only earn a fraction  $(1 - \tau_{t+1})$  of  $R_{t+1}^e$ , where I explain  $\tau_{t+1}$  and  $R_{t+1}^e$  in the next section. This assumption captures an un-modeled lack of skills by financial firms to manage non-financial business when compared to entrepreneurs. Since loan brokers write contracts ensuring that financial firms invest in their funds, I proceed using this implication.

I allow different financial firms to experience different returns on their assets. In period t+1, after earning  $R_{t+1}^f$  by investing in loan brokers, each financial firm *i* draws an idiosyncratic rate of return  $\omega_{i,t+1}^f$ , thus, making its total return on assets be  $\omega_{i,t+1}^f R_{t+1}^f$ . I assume that  $\omega_{i,t+1}^f$  is only observed by firm *i* and is i.i.d. across firms with cdf  $F_{t+1}^f$  and log-normal distribution with  $\mathbb{E}(\omega_{i,t+1}^f) = 1$  and  $\operatorname{Var}\left(\log\left(\omega_{i,t+1}^f\right)\right) = \sigma_{t+1}^f$ . Additionally,  $\log(\sigma_{t+1}^f)$  follows an auto-regressive process of order 1.

It is important to discuss the interpretation for  $\omega_{it}^{f}$  and  $\sigma_{t}^{f}$  in the context of the financial sector. The idiosyncratic rate of return  $\omega_{it}^{f}$  is meant to represent the diversification achieved by financial firms in their assets. In reality, different financial intermediaries use different credit intermediation pratices (e.g. local market knowledge, credit scores) and lend in different markets (e.g. credit cards, business loans or asset backed securities). The combination of all these factors imply in distinct asset rentabilities. Following the same line of thought,  $\sigma_{t}^{f}$  represents the extent to which different financial firms achieve different asset diversifications. For instance, a decrease in  $\sigma_{t}^{f}$  could represent the convergence of banking practices among different financial firms<sup>3</sup>. Another environment change with the same implication is the development of interbank markets. With a bigger array of financial instruments available, financial firms are better able to insure themselves against a possible concentration of their investments, thus, reducing the dispersion of returns among them. On the other hand, a failure of such interbank market, like the one that occurred during the financial crisis of 2008, might have lead to a less diversified financial sector, thereby increasing the dispersion of

<sup>&</sup>lt;sup>3</sup>The generalized investments in computer hardware and software, together with new financial engineering techniques and credit databases during the 1990's and 2000's, may illustrate this argument.

returns of financial firms.

Mutual funds compete in the market of loans to financial firms by choosing the terms of the contract  $(B_{i,t+1}, Z_{i,t+1}^f)$ . However, it is useful to define  $L_{i,t+1}^f$  and  $\overline{\omega}_{i,t+1}^f$  such that:

$$L_{i,t+1}^{f} = \frac{A_{i,t+1}}{N_{i,t+1}^{f}}, \qquad Z_{i,t+1}^{f} B_{i,t+1} = \overline{\omega}_{i,t+1}^{f} R_{t+1}^{f} A_{i,t+1}$$

It is worth emphasizing some remarks here. First, since there is a one-to-one mapping between  $(L_{i,t+1}^{f}, \overline{\omega}_{i,t+1}^{f})$  and  $(B_{i,t+1}, Z_{i,t+1}^{f})$ , we can look for the optimal contract in terms of  $(L_{i,t+1}^{f}, \overline{\omega}_{i,t+1}^{f})$ . Second,  $L_{i,t+1}^{f}$  represents the leverage of financial firm *i* allowed by the loan contract. Third,  $\overline{\omega}_{i,t+1}^{f}$  defines a threshold determining whether financial firm *i* is able, or not, to pay its debt. More precisely, if  $\omega_{i,t+1}^{f} \geq \overline{\omega}_{i,t+1}^{f}$ , then, under the optimal contract, firm *i* pays the lender the amount owed,  $Z_{i,t+1}^{f}B_{i,t+1}$ , and keeps the rest. If  $\omega_{i,t+1}^{f} < \overline{\omega}_{i,t+1}^{f}$ , then, firm *i* does not own enough resources to pay its debt, therefore, declaring bankruptcy. In this situation, the lender forecloses all the remaining assets of the borrowing firm, pays an auditing cost proportional to these assets and retains  $(1 - \mu^{f}) \omega_{i,t+1}^{f}R_{t+1}^{f}A_{i,t+1}$ , for  $\mu^{f} \in (0, 1)$ .

Since financial firms are risk neutral and only care about their equity, mutual funds seek loan contracts that maximize financial firms' expected earnings:

$$\mathbb{E}_{t}\left(\int_{\overline{\omega}_{i,t+1}}^{\infty} \left(\omega - \overline{\omega}_{i,t+1}^{f}\right) dF_{t+1}^{f}(\omega) \frac{R_{t+1}^{f}A_{i,t+1}}{N_{i,t+1}^{f}}\right) = \mathbb{E}_{t}\left[\left(1 - \Gamma_{t+1}^{f}(\overline{\omega}_{i,t+1}^{f})\right) R_{t+1}^{f}L_{i,t+1}^{f}\right], \quad (1)$$
where
$$G_{t+1}^{f}(\overline{\omega}_{i,t+1}^{f}) = \int_{0}^{\overline{\omega}_{i,t+1}^{f}} \omega dF_{t+1}^{f}(\omega),$$

$$\Gamma_{t+1}^{f}(\overline{\omega}_{i,t+1}^{f}) = (1 - F_{t+1}^{f}(\overline{\omega}_{i,t+1}^{f}))\overline{\omega}_{i,t+1}^{f} + G_{t+1}^{f}(\overline{\omega}_{i,t+1}^{f}),$$

and the normalization of (1) by  $N_{i,t+1}^{f}$  can be done because it is taken as given by the contract.

In order to finance its loans, mutual funds issue non-contingent debt to households at the riskless interest rate  $R_{t+1}$ . However, these mutual funds do not have access to state contingent claims in addition to those celebrated with the financial firms. As a result, in every contract between mutual funds and financial firms with equity<sup>4</sup>  $N_{i,t+1}^{f}$ , the funds received in each state of nature of period t+1 must be greater or equal to the funds paid to households:

$$(1 - F_{t+1}^{f}(\overline{\omega}_{i,t+1}^{f}))B_{i,t+1}Z_{i,t+1}^{f} + (1 - \mu^{f})G_{t+1}^{f}(\overline{\omega}_{i,t+1}^{f})R_{t+1}^{f}A_{i,t+1} \geq R_{t+1}B_{i,t+1},$$
(2)

where the first term above represents the payoff from financial firms who fully pay their loans, the second, the payoff from bankrupt financial firms and the third, the funds owed to households. If we

<sup>&</sup>lt;sup>4</sup>It is useful to think that there is a unit mass of financial firms for every level of equity  $N_{i,t+1}^{f}$ , even though it is shown to be an unnecessary assumption.

normalize equation (2) by  $N_{i,t+1}^{f}$  and impose zero profits on mutual funds from the competition in the market of loans, we have:

$$\left[\Gamma_{t+1}^{f}(\overline{\omega}_{i,t+1}^{f}) - \mu^{f}G_{t+1}^{f}(\overline{\omega}_{i,t+1}^{f})\right]R_{t+1}^{f}L_{i,t+1}^{f}/(L_{i,t+1}^{f} - 1) = R_{t+1}.$$
(3)

The contracting problem faced by mutual funds consists in choosing  $(L_{i,t+1}^f, \overline{\omega}_{i,t+1}^f)$  that maximizes (1) subject to (3). Notice that such problem does not depend on the level of equity held by financial firms, and therefore nor does its solution. In turn, this implies that all financial firms hold the same leverage  $L_{t+1}^f$  and face the same threshold  $\overline{\omega}_{t+1}^f$  independently of their equity level.  $N_{i,t+1}^f$ .

After the resolution of both aggregate and idiosyncratic states of nature, two events determine the final amount of equity financial firms retain in order to apply for loans in the following period. First, a mass of  $(1-\gamma^f)$  of firms is randomly selected to start over independently of their equity size. These firms transfer the totality of their assets to households. Just as in BGG, this modeling device allows us to avoid the situation where the financial sector accumulates so much equity that it becomes self-financed, while additionally capturing some of the natural dynamics of death/birth of firms. Second, all financial firms receive a lump-sum transfer of  $W_t^f$  from households. This transfer makes sure that those firms have enough equity to apply for loans in the next period. With these remarks in mind, we have the following law of motion for aggregate financial equity:

$$N_{t+1}^{f} = \gamma^{f} \left[ 1 - \Gamma_{t}^{f}(\overline{\omega}_{t}^{f}) \right] R_{t}^{f} A_{t} + W_{t}^{f},$$
  
where  $A_{t} = \int A_{i,t} di$  and  $N_{t}^{f} = \int N_{i,t}^{f} di.$ 

#### **3.2** Contract between loan brokers and entrepreneurs

There is a unit mass of entrepreneurs. For simplicity, I make assumptions ensuring that entrepreneurs have the same amount of equity whenever they apply for new loans. In the end of period t, they apply for contracts  $(B_{t+1}^e, Z_{t+1}^e)$ , where  $B_{t+1}^e$  is the loan amount and  $Z_{t+1}^e$  is the state-contingent interest rate. With loan  $B_{t+1}^e$  and equity  $N_{t+1}^e$ , entrepreneurs purchase physical capital  $K_{t+1}$  in competitive markets, totaling an amount of assets of  $Q_t K_{t+1} = N_{t+1}^e + B_{t+1}^e$ , where  $Q_t$  is the unit price of capital.

Similar to the case of financial firms, I allow different entrepreneurs to earn different rates of return on their assets. In the beginning of period t + 1, each entrepreneur draws an idiosyncratic shock  $\omega_{t+1}^e$  only observable by himself, which transforms  $K_{t+1}$  into  $\omega_{t+1}^e K_{t+1}$  efficient units of capital. As before,  $\omega_{t+1}^e$  is i.i.d. across entrepreneurs with cdf  $F_{t+1}^e$ , log-normal distribution with  $\mathbb{E}(\omega_{t+1}^e) = 1$ and  $\operatorname{Var}(\log(\omega_{t+1}^e)) = \sigma_{t+1}^e$ , and  $\log(\sigma_{t+1}^e)$  following an auto-regressive process of order 1. Regarding the interpretation of  $\omega_t^e$ , I follow the same idea presented by BGG and CMR: entrepreneurs may have different inventive capabilities. Those obtaining high  $\omega$ 's transform raw capital into successful products (desktop computers, iPhone's,...) while those with low  $\omega$ 's experience unfruitful ideas.

During period t + 1 and with  $\omega_{t+1}^e K_{t+1}$  efficient units of physical capital in hands, entrepreneurs make several decisions, which ultimately lead to the rate of return  $\omega_{t+1}^e R_{t+1}^e$  of purchasing capital. First, they determine the capital utilization  $u_{t+1}$  taking into account the cost function  $a(u_{t+1})$ and the real rental rate of capital  $r_{t+1}^k$ . Second, they rent  $u_{t+1}\omega_{t+1}^e K_{t+1}$  as capital services to intermediate firms and pay the nominal utilization  $\cos a(u_{t+1})P_{t+1}$ . Finally, after goods production takes place, entrepreneurs receive the depreciated capital back from intermediate firms and sell it to households. Summarizing all those transactions, we have that the return on capital for an entrepreneur experiencing a shock  $\omega_{t+1}^e$  is:

$$\omega_{t+1}^e R_{t+1}^e = \omega_{t+1}^e \frac{\left(u_{t+1}r_{t+1}^k - a(u_{t+1})\right)P_{t+1} + (1-\delta)Q_{t+1}}{Q_t}.$$

Notice that the optimal capital utilization is independent of  $\omega_{t+1}^e$  and is pinned down by first order condition  $r_{t+1}^k = a'(u_{t+1})$ .

Analogously to the contracting problem of the previous section, the realization of  $\omega_{t+1}^e$  determines whether the entrepreneur is able to meet his debt obligations or not. If  $\omega_{t+1}^e R_{t+1}^e Q_t K_{t+1} \ge Z_{t+1}^e B_{t+1}^e$ , then the entrepreneur pays his debt,  $Z_{t+1}^e B_{t+1}^e$ , and keeps the rest of his assets. Otherwise, he declares bankrupcy and surrenders all of his assets to the lending loan broker, who pays an auditing cost proportional to the realized gross payoff of the borrowing entrepreneur:  $\mu^e \omega_{t+1}^e R_{t+1}^e Q_t K_{t+1}$ , where  $\mu^e \in (0, 1)$ . Additionally, I assume that loan brokers face operational costs  $\theta_t$  proportional to the loan amount  $B_{t+1}^e$  they lend, where  $\theta_t = \theta(R_t^e, \sigma_t^e)$ ,  $\theta_{1,t} < 0$  and  $\theta_{2,t} > 0$ . This is meant to capture how the costs of knowing the distribution of entrepreneurs  $F_{t+1}^e$  decrease when entrepreneurs are, on average, earning high rates of return and do not have a high dispersion of possible outcomes. Also, I define  $\overline{\omega}_{t+1}^e$  such that  $Z_{t+1}^e B_t^e = \overline{\omega}_{t+1}^e R_{t+1}^e Q_t K_{t+1}$ , where its interpretation is analogous to the one associated with  $\overline{\omega}_{t+1}^f$ .

Since entrepreneurs are risk neutral and seek to maximize their total equity, loan brokers look for loan contracts maximizing entrepreneurs' expected earnings, where the latter is an expression analogous to equation (1). Additionally, these brokers need to guarantee that financial firms are at least indifferent between investing in their funds and opening up their own business. This is accomplished by offering state contingent rates<sup>5</sup>  $R_{t+1}^f = (1 - \tau_{t+1})R_{t+1}^e$  to financial firms, where

<sup>&</sup>lt;sup>5</sup>Another way to understand this requirement is by thinking about the contract written by the loan brokers as one satisfying a participation constraint for financial firms.

 $\tau_{t+1} = \tau \left(\sigma_{t+1}^e\right)$  and  $\tau'(\cdot) > 0$ . I let  $\tau$  depend on  $\sigma_{t+1}^e$  to capture a lower willingness of financial firms to switch sectors when the uncertainty about the returns of the non-financial sector is higher<sup>6</sup>. Again, I assume that loan brokers do not have access in period t to state-contingent markets securities other than the ones offered to financial firms. As a result, the funds received in each period t + 1 state of nature must be no less than the funds paid to financial firms in that state of nature:

$$(1 - F_{t+1}^{e}(\overline{\omega}_{t+1}^{e}))B_{t+1}^{e}Z_{t+1}^{e} + (1 - \mu^{e})G_{t+1}^{e}(\overline{\omega}_{t+1}^{e})R_{t+1}^{e}Q_{t}K_{t+1} - \theta_{t+1}B_{t+1}^{e} = R_{t+1}^{f}B_{t+1}^{e} \left[\Gamma_{t+1}^{e}(\overline{\omega}_{t+1}^{e}) - \mu^{e}G_{t+1}^{e}(\overline{\omega}_{t+1}^{e})\right]R_{t+1}^{e}Q_{t}K_{t+1} - \theta_{t+1}B_{t+1}^{e} = R_{t+1}^{f}B_{t+1}^{e}$$
(4)

where the intuition for the equations above is analogous to that of equations (2) and (3).

Since I impose assumptions guaranteeing that all entrepreneurs apply for loans with the same amount of equity, the contract  $(B_{t+1}^e, Z_{t+1}^e)$  must be the same for all entrepreneurs. Additionally, given that all the funding available for the loan brokers comes from financial firms, market clearing implies that  $B_{t+1}^e = A_{t+1}$ . Finally, competition in the loans market implies that the state contingent schedule of interest rates  $Z_{t+1}^e$  (or  $\overline{\omega}_{t+1}^e$ ) has to be the lowest one satisfying equation (4). These last two conditions pin-down the contract  $(B_{t+1}^e, Z_{t+1}^e)$ .

The determination of the aggregate equity of the entrepreneurial sector is analogous to the one of the financial sector. After the resolution of both the aggregate and idiosyncratic uncertainty, a fraction  $\gamma^e$  of entrepreneurs is randomly selected independently of equity size. These entrepreneurs transfer all of their equity to households. Then, all entrepreneurs return to the household and a new one unit measure of agents is randomly selected to become entrepreneurs. They inherit all the aggregate equity left by the previous entrepreneurs and a lump-sum transfer  $W_t^e$ , sharing these equally among themselves. Then, we have the following law of motion for entrepreneurial equity:

$$N_{t+1}^e = \gamma^e \left[1 - \Gamma_t^e(\overline{\omega}_t^e)\right] R_t^e Q_{t-1} K_t + W_t^e$$

## 4 Model implications for empirical analysis

With the increasing interest in the effects of "uncertainty shocks", and with a myriad of empirical "uncertainty measures" available in the literature, it is worth asking why should we look at some specific measure and not others at any particular context. In the first part of this section, I use the contracting framework of Section 3 to derive the counterpart in the model of the empirical standard deviation of stock returns used in Section 2. Then, I show how this volatility measure is informative about the volatility shocks studied in this paper.

<sup>&</sup>lt;sup>6</sup>This could be rationalized by supposing the existence of fixed/irreversible costs to incur in such switch.

Since this derivation essentially shows that the standard deviation of returns across firms is an endogenous variable in the model, this allows us to explore its response to any structural shock. In particular, in Section 4.2, I study the reaction of several model variables, including financial and non-financial volatilities, to *financial* and *non-financial* volatility shocks. Then, I show that by focusing only on the signs of these reactions, I am able to distinguish a financial volatility shock from a non-financial one, as well as from many other shocks used in the literature. This result is important because it allows me to use these signs to identify the financial volatility shocks in the data.

The derivation of Section 4.1 has other important implications. By showing that the volatility measure across firms is an endogenous variable, it demonstrates that it is not the case that only volatility shocks influence the standard deviation of returns over time. It allows the possibility of structural shocks other than the volatility ones to account for the variation of the financial and non-financial volatilities. Additionally, this derivation formalizes the use of these standard deviation of returns in the estimation of a wide class of DSGE models. This is important because it provides an additional restriction on the estimation of model parameters and contribution of shocks to macroe-conomic fluctuations. Among the models that could use volatility measures in their estimation are BGG, Christiano, Motto and Rostagno (2013) (henceforth CMR), Nolan and Thoenissen (2009) and Hirakata et al (2011).

### 4.1 Volatility measures

During this section, I omit the time subscripts in order to ease the burden of notation. Additionally, I focus on derivation for the case of financial firms. The extension for entrepreneurs is analogous to the one done below. Define  $N'_i$  as financial firm's *i* equity after the realization of the aggregate and idiosyncratic states of nature, but before equity transfers to and from households. Define also  $R^N_i = \frac{N'_i}{N_i}$  as the realized return on financial firm's *i* equity and  $[x]^+ = \max\{x, 0\}$ . Finally, remember that  $Z_i B_i = \overline{\omega}^f R^f A_i$ , where  $B_i$  is the loan amount lent to financial firm *i*,  $Z_i$  is the interest rate charged on the loan,  $A_i$  is *i*'s total amount of assets,  $R^f$  is the aggregate return on assets for financial firms and  $\overline{\omega}^{f}$  is the bankrupcy threshold on the idiosyncratic return rates  $\omega_{i}$ . Then, we have:

$$N'_{i} = \begin{cases} \omega_{i}R^{f}A_{i} - Z_{i}B_{i}, & \text{if } \omega_{i}R^{f}A_{i} \ge Z_{i}B_{i} \\ 0, & \text{otherwise} \end{cases}$$
$$N'_{i} = \left[\omega_{i} - \overline{\omega}^{f}\right]^{+}R^{f}A_{i}$$
$$R^{N}_{i} = \left[\omega_{i} - \overline{\omega}^{f}\right]^{+}R^{f}\frac{A_{i}}{N_{i}}$$
$$R^{N}_{i} = \left[\omega_{i} - \overline{\omega}^{f}\right]^{+}R^{f}L^{f}, \qquad (5)$$

where  $L^f$  is a leverage measure. The independence of  $L^f$  with respect to the level of both equity  $N_i$ and return  $\omega_i^f$  is explained in Section 3.1 and is also present in many other models such as BGG.

Ideally, one would like the financial firms in the model to represent any real financial firm participating in a debt contract. However, due to the need for data on equity returns frequently updated over time (monthly/quarterly), I focus on firms with stocks being traded in the financial markets. More precisely, I proxy the model's equity return for non-bankrupt firms (i.e firms *i* with  $R_i^N > 0$ ) by observable stock returns from the CRSP database. This procedure takes care of the fact that bankrupt firms also do not have stocks being traded in financial markets.

Since we are interested in variables measuring the overall state of the economy and not in the behavior of a particular firm, it is useful to aggregate the information provided in equation (5). For this, remember that  $\omega_i$ 's for financial firms are assumed to have a common distribution. Thus, we can use the statistical moments of returns  $R_i^N$  across firms to provide additional links between the model and the data. Focusing on the first two moments of equation (5) for non-bankrupt firms, we have that the average equity return among financial firms  $R^{N,f}$  and the standard deviation of these equity returns  $\text{Std}^f$  can be expressed as:

$$R^{N,f} = \mathbb{E}\left(R_i^N | R_i^N > 0\right) = R^f L^f \mathbb{E}\left(\omega_i | \omega_i \ge \overline{\omega}^f\right),\tag{6}$$

$$Std^{f} = Std\left(R_{i}^{N}|R_{i}^{N}>0\right) = R^{f}L^{f}Std\left(\omega_{i}|\omega_{i}\geq\overline{\omega}^{f}\right),$$
(7)

where the expectation  $\mathbb{E}(\cdot)$  and standard deviation  $\operatorname{Std}(\cdot)$  operators are taken with respect to the  $\omega_i$ 's of financial firms in each aggregate state of nature. Given a specific distribution for  $\omega_i$ , we can find equations (6) and (7) depending on specific functional forms. For the log-normal case, I provide the derivation in Appendix B. Additionally, since one could derive an equation analogous to (5) for entrepreneurs, we also have equations analogous to (6) and (7) for the non-financial sector. Thus, define  $\mathbb{R}^{N,e}$  as the average equity return among entrepreneurs and  $\operatorname{Std}^e$  as the standard deviation of these entrepreneural returns.

It is worth noticing that, in the calculation of (6) and (7), two firms, with different equity sizes but with the same return  $R_i^N$ , receive the same weight. The intuition for this comes from the fact that these firms draw their  $\omega$ 's from the same distribution, and therefore should be equally informative about it<sup>7</sup>. Additionally, this provides economic foundations for why I use *un-weighted* returns in my baseline measures of average and standard deviation of returns for both the financial and non-financial sectors. I have also used data measures weighted by firms' market capitalization and the results are almost identical to those presented in Section 6.

It is important to emphasize that equation (7) provides the theoretical structure showing how the volatility measures used in this study relate to the volatility shocks  $\sigma^h$ , for  $h \in \{e, f\}$ . First, notice that the shock  $\sigma^h$  exerts a direct influence in the variation of Std<sup>h</sup>. This comes from the fact that  $\sigma^h$  directly impacts Std  $(\omega_i | \omega_i \geq \overline{\omega}^h, i \in h)$ , and therefore the volatility of equity returns Std<sup>h</sup>. However, it is not true that every observable change in Std<sup>h</sup> corresponds to a movement from the associated volatility shock  $\sigma^h$ . The variables  $L^h$ ,  $R^h$  and  $\overline{\omega}^h$  provide an additional source of variation<sup>8</sup> for Std<sup>h</sup>. Moreover, if some DSGE model (such as the one in Section 3) is the true source of the data,  $L^h$ ,  $R^h$  and  $\overline{\omega}^h$  are endogenous variables. In turn, this means that Std<sup>h</sup> is also an endogenous variable, and consequently, any time series analysis of the volatility of returns Std<sup>h</sup> starts from the usual challenge of disentangling its exogenous from its endogenous sources of variation.

Another implication of equations (6) and (7) is that they let us incorporate data on average and standard deviation of equity returns into time series analysis based on models using BGG-like contracts (such as the one studied here) in a precise manner. For instance, if a researcher wants to estimate a DSGE model such as BGG or CMR, (6) and (7) provide additional observation equations further restricting the range of possible estimated shocks and parameters. On the other hand, if the objective is to identify shocks through the estimation of vector autoregressions and imposition of sign restrictions on impulse response functions, models such as the one from Section 3 are able to

<sup>7</sup>One may argue that these conclusions are also dependent on the fact that all firms from the same sector have the same leverage in the model. However, suppose there is another model in which both  $\omega_i$ 's and  $L_i$ 's are allowed to vary across firms while having the same  $\overline{\omega}$  and distribution of  $\omega$ . In this case, it is useful to write equation (5) as  $\frac{R_i^N}{L_i} = [\omega_i - \overline{\omega}]^+ R^f$ . This means that we should weigh returns  $R_i^N$ 's by the inverse of firm's *i* leverage in order to find informative statistics about the distribution of  $\omega_i$ . Since it is well known that, in observable data, leverage is increasing with firm's size, in the absence of data on leverage, the weights on  $R_i^N$  should be decreasing in equity's size, and not the opposite. The intuition for this result comes from the fact that leverage  $L_i$  enhances the rentability of assets  $\omega_i R^f$  in order to provide the final return on equity. Therefore, if the goal is to uncover the return on assets, one should discount the effect from such leverage.

<sup>8</sup>I have also used volatility measures where I divide the standard deviation of returns by the associated sectoral (financial and non-financial) leverage from the Flow of Funds. The results were very similar to those presented in Section 6.

give us precise predictions on the reactions of  $\mathbb{R}^{N,h}$  and  $\mathrm{Std}^h$ , for  $h \in \{f, e\}$ , to any structural shock.

### 4.2 Effects of volatility shocks

In this section, I characterize the signs of the concurrent reactions of several model variables to financial and non-financial volatility shocks. I focus on conclusions as robust as possible to the calibration of the model. In order to do so, I conduct a simulation exercise designed to evaluate whether the signs of these reactions to volatility shocks is consistent across a credible set for the parameters of the model. For more details about this simulation, see Appendix E.2.

The first conclusion from these simulations is that both volatility shocks imply similar concurrent responses for many variables. The intuition for this is twofold. First, since both of them are recessionary shocks, they imply a fall on many macroeconomic variables, such as GDP, investment, hours worked, inflation and the risk free rate. Second, these disturbances are the origin of a credit supply shock: there is a greater mass of borrowers to which it is riskier to lend, and therefore credit falls and funding rates rise throughout the economy.

In this context, it becomes fruitful to turn our attention to the volatility of returns across financial,  $\operatorname{Std}_t^f$ , and non financial,  $\operatorname{Std}_t^e$ , firms. The key result is that different volatility shocks ( $\sigma_t^f$  and  $\sigma_t^e$ ) have different implications for different volatilities of returns ( $\operatorname{Std}_t^f$  and  $\operatorname{Std}_t^e$ ). In order to understand this, remember that  $\operatorname{Std}_t^h$  (for  $h \in \{e, f\}$ ) has three components: leverage  $L_t^h$ , sectoral rate of return on assets  $R_t^h$ , and the standard deviation of idiosyncratic asset returns of non-bankrupt firms  $\operatorname{Std}(\omega_t^h|\omega_t^h \geq \overline{\omega}_t^h)$ . The first is a pre-determined variable, which is a consequence of the idea that equity return comes from the investment of both loans and equity held in the previous period. Therefore, simultaneously to any exogenous shock to the economy, leverage  $L_t^h$  does not contribute to the variation of  $\operatorname{Std}_t^h$ . The second term,  $R_t^h$ , decreases for both financial (h = f) and non-financial (h = e) firms concurrently to both volatility shocks  $\sigma^h$ , for  $h \in \{e, f\}$ . The reason is that a lower amount of resources allocated to investment depresses the price and return on capital.

The standard deviation of idiosyncratic asset returns,  $\operatorname{Std}(\omega_t^h|\omega_t^h \geq \overline{\omega}^h)$ , is the component of the across-firms volatilities  $\operatorname{Std}_t^h$  with distinct responses to different volatility shocks. To grasp the intuition for this, notice that its variation has two direct sources: the threshold  $\overline{\omega}_t^h$  and the exogenous volatility shock  $\sigma_t^h$ . Under any of these shocks,  $\overline{\omega}_t^h$  always rises for both types of firms. This happens in order to increase the interest rate paid to lenders, thus, compensating them for the losses from a higher mass of bankrupt borrowers. This, in turn, decreases  $\operatorname{Std}(\omega_t^h|\omega_t^h \geq \overline{\omega}^h)$  for  $h \in \{e, f\}$ . On the other hand, a movement of the volatility shock  $\sigma_t^h$  only moves the associated  $\operatorname{Std}(\omega_t^h|\omega_t^h \geq \overline{\omega}^h)$ .

Moreover, it does so by increasing the latter in a way that more than compensates the effect from  $\overline{\omega}_t^h$ . In sum, the net effect of a financial volatility shock is to increase  $\operatorname{Std}\left(\omega_t^f | \omega_t^f \geq \overline{\omega}^f\right)$  and to decrease  $\operatorname{Std}\left(\omega_t^e | \omega_t^e \geq \overline{\omega}^e\right)$ , while the opposite holds for non-financial volatility shocks. It is also important to point out that under the event of a shock  $\sigma_t^h$ , the increase in  $\operatorname{Std}\left(\omega_t^h | \omega_t^h \geq \overline{\omega}^h\right)$  is much larger than the decrease<sup>9</sup> in  $R_t^h$ .

Putting together all the effects above, we have that: a financial volatility shock  $\sigma_t^f$  concurrently increases the financial volatility  $Std_t^f$  and decreases non-financial volatility  $Std_t^e$ , and vice-versa. Moreover, defining  $\downarrow$  as negative response and  $\uparrow$  as positive one, the financial and non-financial volatility shocks imply the following concurrent effects:

						Variables				
Shock	GDP	Investment	Hours worked	Inflation	Risk-free rate	Credit to non-financial firms	Funding cost non-financial	Non-financial volatility	Funding cost financial	Financial volatility
Financial	↓	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	Ļ	$\uparrow$	<mark>↑</mark>
Non-financial	↓	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$	<mark>↑</mark>	$\uparrow$	↓

Table 2: Immediate effects from volatility shocks

I have also inserted many other shocks into the model and none of them imply the immediate effects described in Table 2. Among those, are shocks on: price markup, monetary policy, price of investment goods, government spending, transitory technology, consumption preference and marginal efficiency of investment from CMR(2013); and on equity from Nolan and Thoenissen (2009). Moreover, even if we transform parameters  $\mu^e$  and  $\mu^f$  into shocks, they still do not imply the concurrent reactions described above. There are many reasons for why this is the case. However, the most common among all these shocks is that they imply the same sign of reaction for both financial,  $\text{Std}_t^f$ , and non-financial volatilities,  $\text{Std}_t^e$ . This comes from the fact that once the acting shock is neither  $\sigma_t^f$  nor  $\sigma_t^e$ , the immediate movement of  $\text{Std}^f$  and  $\text{Std}^e$  come from their sectoral returns  $R_t^f$  and  $R_t^e$ , which move together for all the shocks listed here.

<sup>&</sup>lt;sup>9</sup>Since the dynamics of the class of models analyzed here is heavily dependent on the amount of equity available to potential borrowers and they hold leveraged positions, small movements in  $R_t^h$  are sufficient to generate realistic movements in key macroeconomic variables such as GDP and investment.

## 5 Empirical framework and data

The purpose of this section is to present a framework capable of empirically identifying volatility shocks and answer questions (i)-(iv) outlined in the Introduction. In order to do so, I follow a procedure that allows me to merge the flexibility of vector auto-regressions (VARs) with the theoretical analysis presented in Sections 3 and 4. The procedure consists in estimating reduced form VARs and identifying the volatility shocks by placing sign restrictions on how the economy immediatly reacts to these shocks.

This approach has several advantages. First, it does not require the identifying assumptions to be so strong such that they fully identify the shock of interest. I am allowed to focus on a set of qualitative implications for the data, possibly only achieving a partial identification of the shock. This is particularly useful because it allows me to use the sign predictions from Section 4.2 that are robust to the calibration of the model. Second, I can choose to use only some of the model predictions for the identification of the shocks, saving some of them for model testing. This allows me, for instance, to test the response of model variables included in the VAR for time periods after the one in which the shock hits the economy. Third, there is no need to make assumptions about shocks other than the ones of interest.

### 5.1 VARs with sign restrictions

We start by supposing that the economy may be modeled by a framework such as the one presented in Section 3 and that its solution can be approximated by the following vector auto-regressive (VAR) model:

$$x_t = c + \mathcal{A}_1 x_{t-1} + \ldots + \mathcal{A}_p x_{t-p} + \mathcal{A}_0 \nu_t, \quad \nu_t \sim i.i.d(0, I_n), \tag{8}$$

where  $x_t$  represents a vector with n observable macroeconomic variables,  $I_n$  is an  $n \times n$  identity matrix,  $\nu_t$  are independent unit-variance<sup>10</sup> structural shocks and  $\mathcal{A}_0$  is a matrix whose columns are the immediate effects of shocks  $\nu_t$  on variables  $x_t$ . Additionally, notice that the reduced form representation of the equation above has  $u_t = \mathcal{A}_0 \nu_t$  as the reduced form shock and  $\Sigma = \mathcal{A}_0 \mathcal{A}'_0$  as the covariance matrix of these latter shocks.

Given the objectives of this study, it is sufficient to identify column vectors **a** of matrix  $\mathcal{A}_0$ describing the imediate effects of volatility shocks on the macroeconomic variables  $x_t$ . For this, I use the partial identification technique proposed by Uhlig (2005) and further developed in Rubio-Ramírez

<sup>&</sup>lt;sup>10</sup>This is just a normalization.

et al (2009). For each value of the parameters  $(c, \mathcal{A}_1, \ldots, \mathcal{A}_p, \Sigma)$ , this technique characterizes the set vectors **a** consistent with the identifying assumptions. The uncertainty about the these parameters is dealt using a Bayesian approach. The full procedure used in following sections is described in detail in Appendix F.

### 5.2 Data

The choice of variables included in  $x_t$  presents the researcher with a trade-off. The higher the number of variables in the VAR, the higher the chance of having observable variables with different implications for different structural shocks, thus, helping in the identification process. On the other hand, increasing the number of variables also increases the number of parameters estimated, which may decrease the precision of the estimates of interest and the power to draw any interesting conclusions.

Having these issues in mind, I build the baseline specification of the reduced form VAR with quarterly data of both macro and financial variables for the period 1980Q1-2010Q4. On the macro side, I include: GDP, investment, hours worked, GDP deflator and the Fed Funds rate. Regarding the financial variables, I proxy the amount of credit to non-financial firms by the liabilities of the non-financial sector from the Flow of Funds, the interest rates charged on funding for non-financial firms by the spread of Baa rates over 10-year Treasury rates, and the interest rates charged on funding for financial firms by the spread of rates on certificates of deposit over T-bill's. Finally, I also include the standard deviation of stock returns of financial and non-financial firms as described in Section 2<sup>11</sup>. Appendix A.2 describes the data used in comprehensive detail.

### 5.3 Identifying assumptions

The assumptions identifying the *financial* and *non-financial* volatility shocks are sign restrictions on the immediate responses of all the variables included in the VAR. These signs are those described in Table 2, where GDP deflator is the variable used for inflation in the VAR, Fed Funds rate for the risk-free rate, liabilities from Flow of Funds of non-financial firms for credit to non-financial firms, Baa spread for funding cost of non-financial firms, certificate of deposits spread for funding cost of financial firms, and standard deviation of stock returns of financial and non-financial firms for financial and non-financial volatilities.

<sup>&</sup>lt;sup>11</sup>In alternative specifications, I have also included many other variables, such as the level of equity and average equity returns for financial and non-financial sectors and the results were very similar. Thus, I kept the VAR with the least number of variables.

## 6 Empirical results

Generally speaking, financial volatility shocks do not account for a major share of macroeconomic fluctuations. However, when they do occur, they result in significant and persistent effects. Moreover, when looking for a specific time period in which such a shock was relevant, I find that between 20% and 40% of several variables during the later portion of the Great Recession can be explained by such shock. Non-financial volatility shocks, on the other hand, by themselves they do not seem to explain a large share of variation in the data.

#### 6.1 What are the effects of financial volatility shocks on the economy?

In order to answer this question, I calculate impulse response functions (IRF's) to a one standard deviation financial volatility shock, where the latter is identified according to the empirical procedure described in Section 5. The results are shown in Figure 5. It is important to emphasize that the first period responses are subject to the sign restrictions described in Table 2, and therefore are either smaller or greater than zero by assumption. However, given these first responses, the values of the IRF's for all the following periods result from iterating the reduced form VAR in a completely unrestricted manner. This means the reaction of the economy to the identified shock does not depend on any a priori assumption and relies only on the estimation procedure of the reduced form VAR and how informative the used data is.

In light of these remarks, it is worth highlighting three preliminary results presenting evidence in favor of the theoretical framework used here. The first is that, empirically, financial volatility shocks only have transitory effects on the non-stationary variables of the VAR (GDP, investment, hours worked and credit), just as predicted by the model. The second is that, with exception of the Fed funds rate and Certificate of Deposits, the IRF's of the calibrated model are almost entirely within the probability intervals reported in Figure 5. This is important because it shows that model is able to provide quantitatively realistic predictions about the effects of financial volatility shocks and these effects are supported by the data<sup>12</sup>. The third result comes from testing a prediction of the first to the second period after a financial volatility shock<sup>13</sup>. This prediction is backed by the data, since

<sup>&</sup>lt;sup>12</sup>It is worth also taking into account that the calibration chosen is not optimized to produce IRF's "closest" as possible to those from the data, where the word "close" refers to some metric such as the one used by Christiano, Eichenbaum and Evans (2005). The calibration used here only pools values reported by many other similar studies.

<sup>&</sup>lt;sup>13</sup>This comes from the fact that, in the model, equity held by non-financial firms decreases faster than the amount of

99.4% of the empirical IRF's satisfy this property. More generally, the importance of these results go beyond just providing evidence supporting the specific theoretical model used in this study. Since this same framework rationalizes the assumptions identifying the studied shocks, it is reassuring to verify that predictions not assumed in empirical procedure are corroborated by the data.

Turning our attention to the quantitative effects of financial volatility shocks on the economy, the results are as follows. First, we observe persistent increases in the volatility measures, with median effects lasting for 7 quarters. Then, we observe a decline in the amount of credit to non-financial firms, reaching its trough only after 2 years at a level 0.7% lower than the initial one. Funding rates surge, with the one charged to non-financial firms experiencing the highest and most persistent increase. These movements reverberate on the production side of the economy with lower GDP, investment and amount of hours worked. These three variables reach their troughs after a year at levels 0.4%, 1.5% and 0.5% lower than before and their medians only come back to the initial level after 11, 9 and 12 quarters. Even looking at the lower 10th percentile of the impulse response functions, we still have that GDP, investment and hours worked take 7, 7 and 9 quarters to come back to their initial level. Finally, there is an aggressive response of the Fed Funds following a pattern similar to the one previously described: it reaches its trough after 1 year and takes at least (on the 10th lower percentile) 10 quarters to reach the level preceding the shock.

### 6.2 Do financial volatility shocks contribute to variations in macro variables?

I answer this question using a forecast error variance decomposition (FEVD) analysis. Essentially, this procedure quantifies how much of the mistakes incurred when predicting the VAR variables are due to financial volatility shocks. This measure is calculated for prediction horizons ranging from 1 to 12 quarters ahead and are expressed in percentage of the total variance of the prediction error of the associated horizon. The results are reported in Figure 6.

The general conclusion is that financial volatility shocks account for a significant, though not major, share of the variation of the variables included in the VAR. Focusing on the median FEVD values at the horizons most explained by these financial shocks, we observe explanatory shares ranging from 10% to 30%. Consistent with the associated IRF's, financial volatility shocks reach their largest share in the FEVD of the non-stationary variables (GDP, investment, hours worked and credit) approximately around the same time of the troughs of the IRF's of these variables. Regarding

credit borrowed, thus, increasing their leverage in the following period. This prediction is also robust to the calibration of the model.

the remaining ones, the share weakly increases with the prediction horizon.

However, it is important to point out that, with the exception of the volatility measures and the Certificate of Deposits spread, the FEVD's of most VAR variables are subject to a high degree of uncertainty. This can be seen by the fact that the distance between the 10<sup>th</sup> and 90<sup>th</sup> quantiles range between 20% and 50%. Given this, another way to proceed is to focus on the lower 10th percentile curves. This gives us an idea of the "worst case scenario" in terms of the contribution of the financial volatility shock to the analyzed variables. By doing so, we notice that, with the exception of credit to non-financial firms and inflation, financial volatility shocks still account for approximately 5% to 12% of the variation of the VAR variables.

One important consequence of the results above is that shocks other than the financial volatility one account for the majority of the variation of the financial volatility measure. This conclusion also extends to the non-financial volatility measure and its associated shock, which can be seen in the FEVD reported in Figure 12. These results mean that the volatility measures should not be taken as approximate proxies for their exogenous components. Moreover, identification strategies using Choleski decompositions might be unreliable, specially if they order the volatility variable as first in the VAR. This is so because the underlying assumption of such strategy is exactly that most of the variance of the one-period ahead prediction error of the volatility measure is explained by its volatility shock <sup>14</sup>.

### 6.3 When have financial volatility shocks been relevant?

In order to evaluate the importance of volatility shocks for the American economy in specific time periods, I implement a historical decomposition of the variables used in the VAR. This means that, in each quarter of the sample, each observation is decomposed into three parts: a deterministic trend, a component accounting for the analyzed shocks (financial, non-financial, or both) and another accounting for all the other shocks driving the VAR but not identified here. For the details on this procedure, see Appendix F.1. This decomposition is specially useful because, by putting together the trend and the financial volatility shock components, I am able to describe how would the American economy have behaved if only the financial shock had impulsed it during the sample period.

Following the peaks of the standard deviation of returns of financial firms showed in Figure 1a,

<sup>&</sup>lt;sup>14</sup>Although using many different uncertainty measures and monthly data, there are many papers estimating the effects of uncertainty shocks with identification strategies based on Choleski decompositions. Among those, Bachmann et al (2010) and Alexopoulos and Cohen (2009) order the uncertainty measure as the first variable in the VAR.

I focus on the last three economic downturns. The Great Recession leads to the strongest results. For the sake of exposition, in all historical decompositions done in this paper (Figures 4d, 7, 8, 9, 13), I add the median of the deterministic trend associated to each variable to the analyzed shock components. Therefore, in order to evaluate if the latter is different from zero, we should verify whether or not its probability intervals contain the trend, instead of the zero line.

Figure 7 shows the historical decomposition of the Great Recession. It supports the general view that although events originated in the financial sector did not cause such downturn in the first place, they definitely contributed to push the US economy into an even deeper recession. This can be seen by the fact that even though the probability intervals of the financial volatility shock components include the trends for all variables up until 2008Q2, there is a sharp movement away from these trends around the third and fourth quarter of 2008, a period in which the financial crisis was specially acute. Another evidence supporting this view comes from the size of the financial volatility shocks during this period. Figure 3b, for instance, shows that the financial shock realized at the fourth quarter of 2008 had a median size of 3 standard deviations. On the other hand, for quarters before 2008Q3 and after 2008Q4, it is not possible to state whether such financial shock was different from zero using the 90<sup>th</sup> and 10<sup>th</sup> quantiles of the associated distribution.

Table 3: Historical decomposition of Great Recession, financial volatility shock

Variable	Real GDP	Investment	Hours worked	Credit to non-financial firms	Baa spread	Non-financial volatility	Certificate of deposit spread	Financial volatility
Significant periods	08Q3-09Q1	08Q3-09Q1	08Q3-09Q2	09Q3-09Q4	08Q4-09Q2	09Q1-09Q3	08Q4	08Q4-09Q2
Share explained	38.1%	41.2%	36.6%	31.1%	26.9%	28.9%	19.0%	36.2%

It is also important to quantify the contribution of financial volatility shocks to the Great Recession. Table 3 presents statistics accomplishing this objective. The row "Significant periods" lists the quarters at which the financial volatility shock component is different than zero with 90% probability, while the row "Share explained" shows the average contribution of the financial shock component to the observed data as a percentage deviation from the trend during these "Significant periods". The conclusion of this exercise is that financial volatility shocks account for a sizable percentage of the 2008 recession. This can be seen by the considerable explained shares of the falls of GDP growth (38.1%), investment growth (41.2%), hours worked growth (36.6%) and credit growth (31.1%).

It is also worth asking how much of the monetary policy conducted over the 2007-2010 period can be accounted by the financial volatility shock. On one hand, Figure 7 shows that we cannot state whether such financial shock contributed to the *levels* of the Fed Funds rate over the period. On the other hand, if we look at the *change* in such benchmark rate from one quarter to the next, we have a slightly different result. Figure 3a shows that half of the 1.5% average drop in the last quarter of 2008 was due to the financial volatility shock<sup>15</sup>. This last result is in line with the FOMC meeting statements of the period in which "... the intensification of financial market turmoil ..."<sup>16</sup> is highlighted as a concern and "... the focus of the Committee's policy going forward [was to] support the functioning of financial markets ..."<sup>17</sup>

When looking at the other two recessions, the contribution of the financial volatility shock is much less pronounced. This gives support to the claim that the last downturn was different from the previous ones. Figure 8 displays the historical decomposition for the early 2000's recession. There, the pattern is the absence of any contribution of the financial shock. More precisely, the probability intervals for the shock component on all variables do not move away from their trends in almost any quarter. Figure 9 shows the results for the recession of the early 1990's, where we notice an ambiguous pattern. While there seems to be a contribution for the downturn of both GDP and investment, the effect on all the other variables is not clear enough in order to draw a conclusive result.

### 6.4 What is the role of non-financial volatility shocks?

Analogously to the previous analysis, I focus on impulse response functions (IRF's), forecast variance decompositions (FEVD's) and historical decompositions for the non-financial volatility shock. The general message is that, in comparison to the financial shocks, the non-financial ones result in smaller and less persitent effects on the economy, they account for a smaller variation of the variables used in the VAR and they have played a smaller role in previous recessions. Then, we may ask an additional question: what is the influence of *both* volatility shocks on the economy? Jointly, both shocks are

 $<sup>^{15}</sup>$ The intuition for these two different results comes from the fact that, while the draws for the *levels* of the financial volatility shock component of the Fed Funds rate have a high variance, the majority of these draws decrease in the forth quarter of 2008 in a somewhat parallel manner. Therefore, when taking the *change* over time for these draws in *levels*, we end up with a clear decrease in the rates during the 4th quarter of 2008.

<sup>&</sup>lt;sup>16</sup>FOMC meeting statements, October 8 and October 29, 2008.

<sup>&</sup>lt;sup>17</sup>FOMC meeting statement, December 16, 2008.

responsible for a large part of the variation of the VAR variables and explain approximately half of the drop in economic activity during the Great Recession.

Figure 10 shows the IRF's to a one standard deviation non-financial volatility shock both from the estimated VAR and the calibrated model. There, we observe that the empirical IRF's are also consistent with those predicted by the model, resulting in: higher volatility measures, lower credit, higher funding rates for both financial and non-financial firms and lower economic activity. However, the magnitude and persistence of these estimated effects are approximately half of those estimated for financial shocks.

Figure 11 presents the contribution of non-financial volatility shocks to FEVD's of the VAR variables. The median share of these non-financial shocks range from 5% to 15%, half of those attributed to the financial ones. Additionally, the precision of the FEVD's are similar to those of the financial shocks, with distances between the 90<sup>th</sup> and 10<sup>th</sup> percentiles ranging from 20% to 40%. Regarding the role played by non-financial volatility shocks in the historical decompositions of the last three recessions, we only see occasional quarters being influenced by such shock, thus, I omit these results from this study.

 Table 4: Historical decomposition of Great Recession, both volatility shocks

Variable	Real GDP	Investment	Hours worked	Credit to non-financial firms	Baa spread	Non-financial volatility	Certificate of deposit spread	Financial volatility
Significant periods	08Q4-09Q1	08Q4-09Q1	08Q4-09Q2	09Q3-09Q4	08Q4-09Q2	09Q1-09Q2	07Q4,08Q4	09Q1-09Q3
Share explained	45.4%	48.7%	47.6%	44.5%	38.5%	45.1%	27.0%	39.2%

Turning to the *joint* role of both volatility shocks to the American economy, we first analyze the FEVD for the VAR variables pooling both of these shocks. Figure 12 shows the results. There, we see an increase on the level of the median shares of around 5-15% when comparing to the FEVD of only financial shocks (Figure 6). Focusing on specific magnitudes, these shocks, jointly, explain almost 40% of variables measuring economic activity (GDP, investment and hours worked), a result similar to the one estimated by CMR. Another result worth emphasizing is that even by pooling both of the volatility shocks, we still do not explain the majority of the variation of the volatility measures, with implications already discussed in Section 6.2.

When focusing the analysis on the Great Recession, we observe that volatility shocks had a major influence in such downturn. Figure 13 shows the historical decomposition for such period summing both the financial and non-financial shock components. The qualitative conclusions are similar to those described for the financial volatility shocks, with the main effects of these shocks being concentrated around the third and forth quarter of 2008. However the quantification of these effects, shown in Table 4, bring us new results. Approximately 50% of the observed decrease in the growth of GDP, investment, hours worked and credit in the later portion of this recession is jointly explained by both volatility shocks. This is important because it allow us to explain a very debated time period of this last recession with a small number of shocks.

#### 6.5 What are the effects of financial volatility shocks on consumption?

It is also worth analyzing both the model's prediction for the behavior of aggregate consumption and whether the data conforms these predictions. In order to so, I include consumption in the simulation exercise of Appendix E.2 evaluating what is the sign of its concurrent reaction to volatility shocks. As shown in Table 8, this variable does not present a consistent reaction to financial volatility shocks accross the set of model parameters analyzed. On the hand, calibrations taken from estimated models similar to the one used here imply a falling consumption concurrent to a financial volatility shock. The key ingredient explaining these reactions is the movement of real interest rates. Thus, parameter values determining price stickyness and monetary policy become important to determine the consumption path.

With the remarks above in mind, I include aggregate consumption in the VAR estimation and explore two identification strategies. In the first, I do not impose any sign restriction on the immediate reaction of consumption while keeping the same signs as in the baseline estimation for the remaining variables. These assumptions are not enough to pin down the behaviour of consumption after the exogenous shock. This is demonstrated by the empirical IRF in Figure 4a, which shows a tendency of consumption to fall, but with probability intervals too large to reach any definitive conclusion. When I impose a negative reaction of consumption, the results become quantitatively relevant. We observe the IRF of consumption (Figure 4b) being different than zero for at least 5 periods and its median reaching its trough after 4 periods at a level 0.15% below than its initial one. Additionally, the historical decomposition of consumption for the Great Recession shows that the financial volatility shock explains an important share its behaviour. More precisely, it accounts for 24% and 40% of consumption's deviation from its trend during the third and forth quarters of 2008.













#### Figure 3: Historical decomposition, financial volatility shocks

Figure 4: Aggregate consumption, financial volatility shocks







Figure 5. Empirical IRFs are done for a one standard deviation shock. Theoretical IRFs use the baseline calibration discussed in Appendix E.1.



Figure 6: Forecast error variance decomposition, financial volatility shock

Figure 6. The graphs above represent the probability intervals for the function  $FEVD^{(a)}(k)_{i,i}$  for each VAR variable *i* as explained in equation (17), where k is the number of forecasting quarters ahead.





Figure 7. The gray area represents the Great Recession as dated by the NBER. For all the graphs above, I add the median of the trend (i.e. deterministic component) of each variable to the quantiles of the financial volatility shock component. For more details on these definitions, see equation (19). This is done for better visualization of the shock component against the observed data. In order to draw the graphs of Real GDP, Investment, Hours and Credit to non-financial firms, I estimate the VAR in levels, and, then, obtain the distributions for growth rates reported above





Figure 8. The gray area represents the early 2000's recession as dated by the NBER. For all the graphs above, I add the median of the trend (i.e. deterministic component) of each variable to the quantiles of the financial volatility shock component. For more details on these definitions, see equation (19). This is done for better visualization of the shock component against the observed data. In order to draw the graphs of Real GDP, Investment, Hours and Credit to non-financial firms, I estimate the VAR in levels, and, then, obtain the distributions for growth rates reported above .



Figure 9: Historical decomposition of early 1990's recession, financial volatility shock

Figure 9. The gray area represents the early 1990's recession as dated by the NBER. For all the graphs above, I add the median of the trend (i.e. deterministic component) of each variable to the quantiles of the financial volatility shock component. For more details on these definitions, see equation (19). This is done for better visualization of the shock component against the observed data. In order to draw the graphs of Real GDP, Investment, Hours and Credit to non-financial firms, I estimate the VAR in levels, and, then, obtain the distributions for growth rates reported above

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Figure 10: Impulse response functions to a non-financial volatility shock

Figure 10. Empirical IRFs are done for a one standard deviation shock. Theoretical IRFs use the baseline calibration discussed in Appendix E.1.





Figure 11. The graphs above represent the probability intervals for the function  $FEVD^{(a)}(k)_{i,i}$  for each VAR variable *i* as explained in equation (17), where k is the number of forecasting quarters ahead.





Figure 12. The graphs above represent the probability intervals for the function  $FEVD^{(a)}(k)_{i,i}$  for each VAR variable *i* as explained in equation (17), where k is the number of forecasting quarters ahead.

![](_page_38_Figure_0.jpeg)

![](_page_38_Figure_1.jpeg)

Figure 13. The gray area represents the Great Recession as dated by the NBER. For all the graphs above, I add the median of the trend (i.e. deterministic component) of each variable to the quantiles of the *joint* financial and non-financial shock component. For more details on these definitions, see equation (19). This is done for better visualization of the shock component against the observed data. In order to draw the graphs of Real GDP, Investment, Hours and Credit to non-financial firms, I estimate the VAR in levels, and, then, obtain the distributions for growth rates reported above .

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## A Data

### A.1 Stock returns and volatilities

In order to construct the empirical counterparts of equations (6) and (7), I start with the monthly returns from the CRSP database for January 1980 to December 2010. So as to avoid typos and misreports while not losing information about the tails of the distributions at each month, I eliminate observations with returns higher than 150% per month. This leads to a loss of about 2% of the initial data, which is the total amount of observations after eliminating those with missing returns and industry classification. Finally, to find quaterly returns for each firm, I compound their monthly returns inside the quarter. In comparison to the procedure adopted by Bloom (2009), I discard far fewer observations. Bloom (2009) constructs his measure for the period June 1962 - June 2008 eliminating returns in the bottom and top 0.5% percentiles for each month. Additionally, he also disregards firms surviving less than 500 months.

In order to classify the firms among financial and non-financial, I use all the information available in the sample. On one hand, CRSP provides the most recent US Census classification, NAICS, and an older one, SIC. On the other hand, there is a SIC code for all firms while the NAICS is available only for some. Given this, and in order to avoid an outdated classification procedure of an ever changing financial sector, I place an emphasis on the NAICS classification. Moreover, since this study focus on private financial firms, I look for those with the following three-digit NAICS classification: 522 (Credit Intermediation and Related Activities), 523 (Securities, Commodity Contracts, and Other Financial Investments and Related Activities), 524 (Insurance Carriers and Related Activities) and 525 (Funds, Trusts, and Other Financial Vehicles). Having all the above issues in mind, I adopt the following classification procedure:

- (a) for those firms with a NAICS code available, I classify:
  - (a1) as financial those with codes 522, 523, 524 or 525;
  - (a2) as non-financial those with codes other than those above;
- (b) for those firms without a NAICS code, I use information from the US Census website about bridging the two classifications to find the SIC codes associated with the 3-digit-NAICS codes 522, 523, 524 or 525. Then, I follow the procedures (a1) and (a2) above.

### A.2 VAR data

GDP is measured in 2009 dollars. Investment is the sum of gross private domestic investment and personal consumption expenditures of durable goods, each one deflated by its own deflator. Consumption is the sum of personal consumption expenditures of non-durables and services, each one also deflated by its own price index<sup>18</sup>. Hours worked is measured by the index of aggregate weekly hours of production and nonsupervisory employees in all private industries. Credit borrowed by non financial firms is the amount of dollars in credit instruments in the liabilities of the nonfinancial corporate and non-corporate sectors provided in "levels" by the Flow of Funds. This credit measure is deflated by the GDP-deflator. Each one of the previous variables is normalized by the total population over 16 years old and taken its log. Inflation is measured by the log-difference of the GDP deflator. The risk free interest rate is the average of the Fed Fund's rate over the quarter. The funding cost for non-financial firms is the spread between the rates Baa and the 10year Treasury constant maturity. The funding cost for financial firms is the spread between the rates on 3-month certificates of deposit negotiated in the secondary market and the 3-month Treasury bill. The volatility measures across firms are the ones already discussed in the previous section.

## B Log-normal algebra

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In order to find closed form solutions for equations (6) and (7) when  $\omega$  is log-normally distributed, we only need to focus on the terms  $\mathbb{E}(\omega|\omega \geq \overline{\omega})$  and  $\operatorname{Std}(\omega|\omega \geq \overline{\omega})$ . Suppose that  $\omega \sim Ln(m, \sigma^2)$ , and define F as the cdf of the  $\omega$ . Also, remember that  $G(\overline{\omega}) = \int_{\omega \leq \overline{\omega}} \omega dF(\omega)$  and that normalization  $\mathbb{E}(\omega) = 1$  implies that  $m = -\frac{\sigma^2}{2}$ . Then, with such normalization, one can show that:

$$\mathbb{E}\left(\omega|\omega\geq\overline{\omega}\right) = \frac{(1-G(\overline{\omega}))}{(1-F(\overline{\omega}))},$$
  
where  $F(\overline{\omega}) = \Phi\left(\frac{\log\left(\overline{\omega}\right)}{\sigma} + \frac{\sigma}{2}\right), \qquad G(\overline{\omega}) = \Phi\left(\frac{\log\left(\overline{\omega}\right)}{\sigma} - \frac{\sigma}{2}\right),$ 

 $\Phi(\cdot)$  is the cdf of a standard normal and the representations for  $F(\cdot)$  and  $G(\cdot)$  result from tricks of change of variables of integration analogous to those shown below.

In order to calculate  $\operatorname{Std}(\omega|\omega \geq \overline{\omega})$ , the only information missing is  $\mathbb{E}(\omega^2|\omega \geq \overline{\omega})$ . Focusing on

<sup>&</sup>lt;sup>18</sup>I could also have followed Smets and Wouters (2007) and deflated the investment and consumption variables by the GDP deflator. I implemented such procedure and the results are very similar to those shown in the paper. While this approach avoids the positive trend in the investment share of output when each variable is deflated by its own price index, it discards the sectoral information specific to each different deflator.

the latter expression, I proceed under a general value for m. Later, I impose normalization  $m = -\frac{\sigma^2}{2}$ . First, notice that:

$$\mathbb{E}(\omega^2|\omega \ge \overline{\omega}) = \int_{\overline{\omega}}^{\infty} \omega^2 \underbrace{\frac{1}{1 - F(\overline{\omega})} \frac{1}{\omega \sigma \sqrt{2\pi}} \exp\left\{-\frac{1}{2} \left(\frac{\ln(\omega) - m}{\sigma}\right)^2\right\}}_{\text{density of lognormal given } \omega \ge \overline{\omega}} d\omega \tag{9}$$

Then, implement  $y = \frac{\ln(\omega) - m}{\sigma}$  as a transformation of variables. This means that:

$$\omega = \exp\{\sigma y + m\}, \qquad d\omega = \sigma \exp\{\sigma y + m\} dy, \qquad \overline{y} = \frac{\ln(\overline{\omega}) - m}{\sigma}.$$

Substituting the three equations above in (9), we have:

$$\begin{split} \mathbb{E}(\omega^2|\omega\geq\overline{\omega}) &= \frac{1}{1-F(\overline{\omega})}\int_{\overline{y}}^{\infty}\exp\left\{\sigma y+m\right\}\frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{1}{2}y^2\right\}\sigma\exp\left\{\sigma y+m\right\}dy\\ &= \frac{\exp\left\{2m\right\}}{1-F(\overline{\omega})}\int_{\overline{y}}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}y^2+2\sigma y\right\}dy\\ &= \frac{\exp\left\{2m\right\}}{1-F(\overline{\omega})}\int_{\overline{y}}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\left(y^2-4\sigma y+4\sigma^2-4\sigma^2\right)\right\}dy\\ &= \frac{\exp\left\{2m+2\sigma^2\right\}}{1-F(\overline{\omega})}\int_{\overline{y}}^{\infty}\frac{1}{\sqrt{2\pi}}\exp\left\{-\frac{1}{2}\left(y-2\sigma\right)^2\right\}dy \end{split}$$

Imposing  $x = y - 2\sigma$  as a new transformation and defining  $\overline{x} = \overline{y} - 2\sigma$ , we have that:

$$\mathbb{E}(\omega^2 | \omega \ge \overline{\omega}) = \frac{\exp\left\{2m + 2\sigma^2\right\}}{1 - F(\overline{\omega})} \underbrace{\int_{\overline{x}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}x^2\right\} dx}_{\operatorname{Prob}(x \ge \overline{x}) \text{ where } x \sim N(0, 1)}$$

If we use normalization  $m = -\frac{\sigma^2}{2}$  and substitute the definition of  $\overline{y}$ , we have that:

$$\overline{x} = \overline{y} - 2\sigma = \frac{\ln(\overline{\omega}) - m}{\sigma} - 2\sigma = \frac{\ln(\overline{\omega})}{\sigma} - \frac{-\frac{\sigma^2}{2}}{\sigma} - 2\sigma = \frac{\ln(\overline{\omega})}{\sigma} - \frac{3}{2}\sigma,$$

and:

$$\mathbb{E}(\omega^2 | \omega \ge \overline{\omega}) = \exp(\sigma^2) \frac{(1 - H(\overline{\omega}))}{(1 - F(\overline{\omega}))}, \quad \text{where} \quad H(\overline{\omega}) = \Phi\left(\frac{\log(\overline{\omega})}{\sigma} - \frac{3}{2}\sigma\right).$$

Finally, we have that:

$$\operatorname{Std}(\omega|\omega \ge \overline{\omega}) = \left\{ \exp(\sigma^2) \frac{(1 - H(\overline{\omega}))}{(1 - F(\overline{\omega}))} - \left[ \frac{(1 - G(\overline{\omega}))}{(1 - F(\overline{\omega}))} \right]^2 \right\}^{1/2}.$$

## C Standard part of the model

### C.1 Goods production

There is a competitive and representative final goods producer who combines intermediate goods  $Y_{jt}$ , for  $j \in [0, 1]$ , to produce a homogeneous good  $Y_t$  using the following Dixit-Stiglitz technology:

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{1}{\lambda_f}} dj\right]^{\lambda_f}, \quad 1 \le \lambda_f < \infty.$$

Intermediate producers purchase capital and labor in competitive markets and use them in the following production function:

$$Y_{jt} = \begin{cases} K_{jt}^{\alpha} H_{jt}^{(1-\alpha)} - \phi & \text{if } K_{jt}^{\alpha} H_{jt}^{(1-\alpha)} > \phi \\ 0 & \text{otherwise,} \end{cases}$$

where  $K_{jt}$  is the amount of capital services rented,  $H_{jt}$  is the amount of homogenous labor hired and  $\phi$  is the fixed cost incurred<sup>19</sup>. These intermediate producers monopolistically set their prices  $P_{jt}$ subject to Calvo-style frictions. More precisely, in each period, a randomly selected fraction  $(1 - \xi_p)$ of these producers is allowed to choose their optimal price, while the remaining  $\xi_p$  fraction follow an indexation rule:  $P_{j,t} = \tilde{\Pi}_t P_{j,t-1}$ . The indexation  $\tilde{\Pi}_t$  is an weighted average between the inflation of the previous period,  $\Pi_{t-1}$ , and the one prevailing in steady state,  $\Pi^{ss}$ :

$$\widetilde{\Pi}_t = (\Pi^{ss})^{\iota} (\Pi_{t-1})^{1-\iota},$$

where  $\Pi_{t-1} = P_{t-1}/P_{t-2}$  and  $P_t = \left[\int_0^1 P_{jt}^{\frac{1}{1-\lambda_f}} dj\right]^{1-\lambda_f}$ . Final goods  $Y_t$  can be transformed into either investment goods  $I_t$ , consumption goods  $C_t$ , or government expenditures  $G_t$ , with a one-for-one technology. Therefore,  $Y_t$ ,  $C_t$ ,  $I_t$  and  $G_t$  have the same unit price  $P_t$  and can be thought as being supplied by the final goods producer.

### C.2 Households

There is a large number of households. They are all identical and able to supply all types of differentiated labor services  $h_{it}$  for  $i \in [0, 1]$ . Within each household, I let its members pool their incomes, thus, adopting the large family assumption of Andolfato (1996) and Merz (1995). These households choose their consumption  $C_t$ , investment  $I_t$ , savings  $B_{t+1}$ , and end-of-period-t physical capital  $K_{t+1}$ , facing competitive markets. The labor supply is subject to Calvo-style frictions and

<sup>&</sup>lt;sup>19</sup>The value of  $\phi$  is chosen to ensure zero profits in steady state for intermediate producers.

is described in detail in the next section. Underlying all households' choices are the following preferences:

$$\mathbb{E}_{0} \sum_{t=0}^{\infty} \beta^{t} \left( \log \left( C_{t} - b C_{t-1} \right) - \psi_{0} \int_{0}^{1} \frac{h_{it}^{1+\psi_{l}}}{1+\psi_{l}} di \right).$$
(10)

After the production of final goods in each period t, households build physical capital  $K_{t+1}$ , and sell it to entrepreneurs at price  $Q_t$ . However, in order to build such  $K_{t+1}$ , they must purchase the existing physical capital from entrepreneurs and investment goods from the final goods producers. The existing capital is just the one produced in the previous period,  $K_t$ , depreciated at rate  $\delta$ . The technology of capital production available to households is:

$$K_{t+1} = (1 - \delta)K_t + (1 - S(I_t/I_{t-1}))I_t,$$
(11)

where  $S(\cdot)$  is an increasing and convex adjustment cost function with S(1) = 0, S'(1) = 0  $S''(1) = \chi > 0$ . Since there is a one-to-one relationship between the depreciated capital,  $(1 - \delta)K_t$ , and the newly produced one,  $K_{t+1}$ , the unit price of the former is also  $Q_t$ .

Finally, the budget constraint of households is:

$$P_t C_t + B_{t+1} + P_t I_t \le R_t B_t + \int_0^1 W_{it} h_{it} \, di + Q_t K_{t+1} - Q_t (1-\delta) K_t + D_t \tag{12}$$

where  $R_t$  is the risk free interest rate paid on households savings,  $W_{it}$  is the nominal hourly wage for each one of the differentiated labor services and  $D_t$  represents all lump sum transfers made to and from households. In short, the problem of the representative household is to choose  $C_t$ ,  $B_{t+1}$ ,  $I_t$  and  $K_{t+1}$ , maximizing (10) subject to (11) and (12). Since the sole source of funding for mutual funds are household savings  $B_t$ , we must have that:  $B_t = \int B_{i,t} di$ .

### C.3 Labor supply

I adopt Erceg, Henderson, and Levin's (2000) wage setting framework. There is a competitive and representative labor aggregator who purchases differentiated labor services  $h_{it}$ , for  $i \in [0, 1]$ , to produce homogeneous labor  $H_t$  using the following Dixit-Stiglitz technology:

$$H_t = \left[\int_0^1 H_{it}^{\frac{1}{\lambda_w}} di\right]^{\lambda_w}, \quad 1 \le \lambda_w < \infty.$$

The labor aggregator sells the produced homogeneous labor to intermediate firms at its unit cost of production  $W_t = \left[\int_0^1 W_{it}^{\frac{1}{1-\lambda_w}} di\right]^{1-\lambda_w}$ .

In order to model the wage setting process, it is useful to suppose the existence of unions representing all household members supplying the same type of differentiated labor  $h_{it}$ . These unions monopolistically sell labor types  $h_{it}$  to the labor aggregator subject to timing restrictions, such as in Calvo (1983). In each period, a randomly selected fraction  $(1 - \xi_w)$  of these unions chooses the optimal wage from the point of view of households. The remaining  $\xi_w$  fraction of unions readjust their wages according to the rule  $W_{it} = \widetilde{\Pi}_{w,t}W_{it-1}$ , where the indexation  $\widetilde{\Pi}_{w,t}$  is a weighted average between the inflation of the previous period,  $\Pi_{t-1}$ , and the one prevailing in steady state,  $\Pi^{ss}$ :

$$\widetilde{\Pi}_{w,t} = (\Pi^{ss})^{\iota_w} (\Pi_{t-1})^{1-\iota_w}.$$

### C.4 Government and resource constraint

The central bank follows the usual Taylor rule:

$$\frac{R_t}{R^{ss}} = \left(\frac{R_{t-1}}{R^{ss}}\right)^{\rho_R} \left[ \left(\frac{\Pi_t}{\Pi^{ss}}\right)^{\alpha_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\alpha_y} \right]^{(1-\rho_R)}$$

where I exclude the usual monetary policy shock since I am not interested in its effects on the economy. Fiscal policy is represented by an exogenous government spending G followed by an equal amount of lump-sum taxes on the household. For simplicity, I also assume that all the monitoring and operational costs incurred by the mutual funds and loan brokers are rebated as lump sum transfers to the household. This captures the idea that both services from auditing bankrupt firms and operations on the financial industry are implemented by a negligible set of specialized agents, who bring those earnings to the realm of consumption smoothing decision. Therefore, I have the following resource constraint:

$$Y_t = C_t + I_t + G.$$

# **D** Functional forms of frictions

In this section, I describe the functional forms of the frictions assumed in the model in order to enrich its dynamics. Function  $S(\cdot)$  is defined as

$$S\left(\frac{I_t}{I_{t-1}}\right) = \exp\left[\sqrt{\frac{\chi}{2}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \exp\left[-\sqrt{\frac{\chi}{2}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] - 2.$$

This adjustment cost specification brings the dynamics aggregate investment closer to what we observe in the data while preserving the same steady state of a frictionless model with  $S(\cdot) = 0$ .

Cost function  $a(\cdot)$  is defined by:

$$a(u) = \frac{r^{k,ss}}{\sigma^a} \left[ \exp\left(\sigma^a(u-1)\right) - 1 \right],$$

and is designed such that  $\sigma^a$  measures the curvature in the cost of adjustment of capital utilization while ensuring u=1 in steady state for any value of  $\sigma^a$ .

Operational costs  $\theta_t$  and return discount  $(1 - \tau_t)$  help with the co-movements of the financial variables in model. Their specification,  $\theta_t = -\theta_1 \log (R_t^e/R^{e,ss}) + \theta_2 \log (\sigma_t^e/\sigma^{e,ss})$  and  $(1 - \tau_t) = \exp \left[-\tau_e \log (\sigma_t^e/\sigma^{e,ss})\right]$ , also do not affect the steady state of the model.

## **E** Model properties

In this section, I discuss the baseline calibration of the model, its implied steady state properties in comparison with the data and the simulation exercise done to show the robustness of the identifying assumptions 5.1 and 5.2 with respect to the model's parameter values.

#### E.1 Baseline calibration and steady state

The baseline calibration is shown in Tables 5 and 6. Table 5 shows the values of the parameters, which are kept fixed throughout the paper. The values of  $\alpha$ ,  $\lambda_f$ ,  $\beta$ ,  $\delta$ ,  $\psi_l$ , and  $\lambda_w$  are taken from Christiano, Eichenbaum and Evans (2005). The values of  $\Pi^{ss}$  and  $G^{ss}/Y^{ss}$  are chosen to be as close as possible to their data counterparts. Parameters  $W^f$ ,  $W^e$ ,  $\gamma^f$  and  $\gamma^e$  contribute to a better fit of the model's steady state to the data, while being close to values chosen by BGG and CMR. The values of  $\theta_1$ ,  $\theta_2$  and  $\tau_e$  are the smallest as possible while delivering sensible co-movements in the model.

The remaining parameters are shown in the "Baseline" column of Table 6. I use the values reported by CMR for  $\rho_R$ ,  $\iota$ ,  $\iota_w$ ,  $\xi_p$ ,  $\xi_w$ , b and  $\mu^e$  while  $\chi$  and  $\sigma^a$  are taken from Christiano, Eichenbaum and Evans (2005). The parameters  $\alpha_y$  and  $\alpha_{\pi}$  are within the range of values reported by the literature. With the absence of references to calibrate  $\mu^f$ , I choose a value within the range defended by Carlstrom and Fuerst (1997) and close to the one reported by CMR for non-financial firms. Regarding  $\sigma^{f,ss}$  and  $\sigma^{e,ss}$ , I select values approximating the model's standard deviation of equity returns for financial firms and entrepreneurs to those observed in the data. Finally,  $\rho_{\sigma f}$  and  $\rho_{\sigma^e}$  are chosen to make the model's IRF's reported in Figures 5 and 10 as close as possible to the IRF's from the data. The steady state implied by baseline calibration is described in Table 7. The values reported are broadly consistent with their empirical counterparts. However, there are two variables for which the model's prediction does not perform very well: the leverage, and the standard deviation of equity returns of financial firms. The intuition for this comes from the contract problem described in Section 3.1 and from the approximation  $\operatorname{Std}^f \approx L^{f,ss} \cdot \sigma^{f,ss}$ . In principle, decreasing  $\sigma^{f,ss}$  would bring down  $\operatorname{Std}^{f,ss}$  if  $L^{f,ss}$  does not increase too rapidly. It turns out that this is true only up to some point, after which the decrease in  $\sigma^{f,ss}$  is exactly compensated by an increase in  $L^{f,ss}$ , making  $\operatorname{Std}^{f,ss}$  reach a minimum still above the data value of 0.18. Thus, I decided to choose a value  $\sigma^{f,ss}$  not too far from those already reported in other models. It is worth noting that the possible non-normality of the distribution of  $\omega's$  could potentially provide an alternative calibration improving this feature of the model. However, for the sake of simplicity and comparability with other models, I chose not to explore this possibility in this study.

### E.2 Robustness of volatility shock effects

The idea of the following simulation exercise is to evaluate whether Assumptions 5.1 and 5.2 hold for a wide range of parameter values of the model. This is accomplished by the following procedure:

- (1) select a set of parameters for which we want to evaluate the robustness of the results;
- (2) choose a set of macroeconomic variables for which we are interested in knowing their imediate reaction to the volatility shocks;
- (3) pin down ranges of plausible values for each one of the parameters in (1);
- (4) draw parameters from a multivariate uniform distribution with support described in (3);
- (5) under calibration (4), compute IRF's of variables in (2) to the volatility shocks
- (6) repeat (4) and (5) for a sufficiently large number
- (7) count the proportion of draws for which the variables in (2) satisfy Assumptions 5.1 and 5.2.

Table 6 lists the set of parameters selected in (1) and their range of possible values. It is worth highlighting that Table 6 includes all parameters estimated by CMR. The ranges of parameter values are selected by pooling the information from several papers, such as CEE, CMR and BGG. It is worth mentioning that the ranges for  $\sigma^{f,ss}$  and  $\sigma^{e,ss}$  are such that the steady state values of  $\text{Std}^{f,ss}$ ,  $L^{f,ss}$ ,  $\text{Std}^{e,ss}$  and  $L^{e,ss}$  do not assume values too far from those observed in the data. Table 8 shows the results of simulation (1)-(7). For this, I repeat procedures (4) and (5) 10,000 times. In short, the simulation displays proportions of 93% or higher in the direction of the Assumptions 5.1 and 5.2, telling us that these assumptions are valid for almost the whole range of parameters described in Table 6. Moreover, focusing only on Assumption 5.1, these proportions are 98% or higher. Expanding the analysis for aggregate consumption shows us one caveat of this exercise. While the majority of the draws indicate that consumption would rise after a volatility shock, this does not mean that these draws are also the most plausible. In fact, using either the baseline calibration of the model (as it is shown in Section 6) or a calibration closer to the one reported by CMR implies a fall in consumption after a volatility shock.

α	Share of physical capital in production function	0.36
$\lambda_{f}$	Steady state markup, intermediate firms	1.46
$\beta$	Preference discount rate	$(1.03)^{-0.25}$
δ	Depreciation rate on capital	0.025
$\Pi^{ss}$	Steady state inflation	$(1.03)^{-0.25}$
$\psi_l$	Curvature on disutility of labor	1
$G/Y^{ss}$	Share of government expenditure of GDP	0.2
$\lambda_w$	Steady state markup, labor supply	1.05
$\gamma^f$	Fraction of exogenous survival of financial firms	0.975
$W^f$	Lump sum transfer to financial firms	$0.005 \cdot N^{f,ss}$
$\gamma^e$	Fraction of exogenous survival of entrepreneurs	0.97
$W^e$	Lump sum transfer to entrepreneurs	$0.008 \cdot K^{ss}$
$ heta_1$	Loan brokers' operational cost elasticity to capital returns	1.03
$\theta_2$	Loan brokers' operational cost elasticity to non-financial volatility shock	0.005
$ au_e$	Elasticity of switch cost to financial firms	0.005
I		

## Table 5: Baseline calibration, quarterly frequency

		Unifo	orm dist	
Parameter	Description of parameter	LB	UB	Baseline
$\rho_R$	Smoothing parameter of interest rate rule	0.6	0.9	0.85
$\alpha_{\pi}$	Weight on inflation in interest rate rule	1.1	2.5	1.5
$\alpha_y$	Weight on GDP growth in interest rate rule	0	0.5	0.3
$\chi$	Curvature of adjustment cost of investment	1	15	3.6
$\sigma^a$	Curvature of adjustment cost of capital utilization	0.01	5	0.01
ι	Weight on steady state inflation of price indexation	0	1	0.9
$\iota_w$	Weight on steady state inflation of wage indexation	0	1	0.49
$\xi_p$	Calvo price stickyness	0.4	0.9	0.74
$\xi_w$	Calvo wage stickyness	0	0.999	0.81
b	Habitat persistence in consumption	0	0.999	0.74
$\mu^{f}$	Auditing cost on financial firms	0.1	0.3	0.2
$\sigma^{f,ss}$	Steady state financial volatility	0.07	0.15	0.1
$\mu^e$	Auditing cost on entrepreneurs	0.2	0.36	0.2
$\sigma^{e,ss}$	Steady state non-financial volatility	0.07	0.15	0.1
$\rho_{\sigma^f}$	Autocorrelation of financial volatility shock	0.5	0.99	0.93
$\rho_{\sigma^e}$	Autocorrelation of non-financial volatility shock	0.5	0.99	0.93

# Table 6: Simulated parameters, uniform distributions and calibration

Variable	Description of Variable	Model	Sample data
$I^{ss}/Y^{ss}$	Ratio of investment to GDP	0.245	$0.21^{1}$
$C^{ss}/Y^{ss}$	Ratio of consumption to GDP	0.555	$0.60^{1}$
$G^{ss}/Y^{ss}$	Ratio of government expenditures to GDP	0.20	$0.198^{2}$
$K^{ss}/Y^{ss}$	Ratio of stock of capital to GDP	10	$10.9^{3}$
$\Pi^{ss}$	Inflation (APR)	3%	$2.86\%^{1}$
$R^{ss}$	Risk free interest rate (APR)	6%	$5.84\%^{1}$
$L^{e,ss}$	Leverage of entrepreneurs	2.42	$1.89^{4}$
$\operatorname{Std}^{e,ss}$	Standard deviation of equity returns of entrepreneurs	0.248	$0.289^{1}$
$L^{f,ss}$	Leverage of financial firms	4	$14.6^{5}$
$\operatorname{Std}^{f,ss}$	Standard deviation of equity returns of fin-firms	0.404	$0.18^{1}$

Table 7: Steady state properties

Notes: <sup>1</sup>The same variables as those used in the VAR estimation. If we deflate investment and consumption variables by the GDP deflator, we have  $I^{ss}/Y^{ss} = 0.25$  and  $C^{ss}/Y^{ss} = 0.55$ . <sup>2</sup>Consumption Expenditures & Gross Investment, source: BEA. <sup>3</sup>Capital stock includes private non-residential fixed assets, private residential, stock of consumer durables and stock of private inventories, source: BEA. <sup>4</sup>The ratio between total assets and total net worth of corporate business, source: Flow of Funds. <sup>5</sup>The ratio between total assets and total assets minus total liabilities of financial business, source: Flow of Funds.

		$\sigma^f_t$ s	hock	$\sigma^e_t$ shock	
Variable	Description of Variable	Reaction	% Draws	Reaction	% Draws
$Y_t$	GDP	$\downarrow$	0.995	Ļ	0.949
$I_t$	Investment	$\downarrow$	1	$\downarrow$	1
$C_t$	Consumption	$\downarrow$	0.387	$\downarrow$	0.242
$H_t$	Hours worked	$\downarrow$	0.996	$\downarrow$	0.955
$\Pi_t$	Inflation	$\downarrow$	0.988	Ļ	0.933
$R_t$	Risk free interest rate	$\downarrow$	0.997	Ļ	0.964
$A_t$	Total credit to entrepreneurs	$\downarrow$	0.987	Ļ	0.964
$Z_t^e$	Funding cost to entrepreneurs	$\uparrow$	1	1	1
$R_t^{N,e}$	Average equity return of entrepreneurs	$\downarrow$	1	Ļ	1
$\operatorname{Std}_t^e$	Standard deviation of entrepreneurs' equity returns	$\downarrow$	1	1	1
$Z_t^f$	Funding cost to financial firms	$\uparrow$	1	1	1
$R_t^{N,f}$	Average equity return of financial firms	$\downarrow$	1	$\downarrow$	1
$\operatorname{Std}_t^f$	Standard deviation of equity returns of fin-firms	↑	1	↓ ↓	0.930

Table 8: Contemporaneous reaction of selected variables to volatility shocks

![](_page_55_Figure_0.jpeg)

Figure 14: Impulse response functions to a financial volatility shock, simulated data

Figure 14. Empirical IRFs are done for a one standard deviation shock. Theoretical IRFs use the baseline calibration discussed in Appendix E.1.

![](_page_56_Figure_0.jpeg)

Figure 15: Impulse response functions to a financial volatility shock, median simulated data

Figure 15. Empirical IRFs are done for a one standard deviation shock. Theoretical IRFs use the baseline calibration discussed in Appendix E.1 .

## **F** Vector autoregressions and sign restrictions

This appendix describes the empirical framework used to answer questions (i)-(iv). Section F.1 keeps parameters  $(c_b, \mathcal{B}_1, \ldots, \mathcal{B}_p, \Sigma, \mathbf{a})$  fixed throughout its analysis while the estimation of these parameters is discussed in Section F.2. The description of the identifying assumptions restricting the set of admissible  $\mathbf{a}$ 's is made in Section 5.3.

### F.1 IRF, FEVD and historical decomposition

Define vector  $\mathbf{a} \in \mathbb{R}^n$  as an *impulse vector* if, and only if, there is a matrix  $\mathcal{A}_0$  such that  $\mathcal{A}_0\mathcal{A}'_0 = \Sigma$ and  $\mathbf{a}$  is a column of  $\mathcal{A}_0$ . In order to proceed, it is useful to present two results from Uhlig(2005). The first establishes that for any matrices  $\mathcal{A}_0$  and  $\tilde{\mathcal{A}}_0$  such that  $\mathcal{A}_0\mathcal{A}'_0 = \Sigma$  and  $\tilde{\mathcal{A}}_0\tilde{\mathcal{A}}'_0 = \Sigma$ , it must be that

$$\mathcal{A}_0 = \mathcal{A}_0 \mathcal{Q},\tag{13}$$

where  $\mathcal{Q}$  is an orthogonal matrix, i.e.  $\mathcal{Q}\mathcal{Q}' = I_n$ . The second result shows that for a fixed  $\tilde{\mathcal{A}}_0$  for which  $\tilde{\mathcal{A}}_0 \tilde{\mathcal{A}}'_0 = \Sigma$ , **a** is an impulse vector if, and only if, there is an unit length vector **q** such that:

$$\mathbf{a} = \tilde{\mathcal{A}}_0 \,\mathbf{q}.\tag{14}$$

The first result provides an efficient way of looking for matrices  $\mathcal{A}_0$  such that  $\mathcal{A}_0\mathcal{A}'_0 = \Sigma$ : choose any matrix  $\tilde{\mathcal{A}}_0$  such that  $\tilde{\mathcal{A}}_0\tilde{\mathcal{A}}'_0 = \Sigma$ , then search on the space of rotations over  $\tilde{\mathcal{A}}_0$ . The second result goes further. It shows that in order to characterize the set of admissible **a** impulse vectors, we do not need to worry about the matrices  $\mathcal{A}_0$  originating vector **a**'s. For a fixed matrix  $\tilde{\mathcal{A}}_0$ , we only need to search for vectors in the linear subspace generated by the columns of  $\tilde{\mathcal{A}}_0$ .

In order to help the presentation of the rest of the framework, I represent equation (8) by its companion form:

$$\begin{bmatrix} x_t \\ x_{t-1} \\ x_{t-2} \\ \vdots \\ x_{t-p+1} \end{bmatrix} = \begin{bmatrix} I_n \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} c_b + \begin{bmatrix} \mathcal{B}_1 & \mathcal{B}_2 & \dots & \mathcal{B}_{p-1} & \mathcal{B}_p \\ I_n & 0 & \dots & 0 & 0 \\ 0 & I_n & \dots & 0 & 0 \\ \vdots & \dots & \ddots & \dots & \dots \\ 0 & 0 & \vdots & I_n & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ x_{t-3} \\ \vdots \\ x_{t-p} \end{bmatrix} + \begin{bmatrix} I_n \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u_t$$

$$X_t = \mathcal{I} c_b + \mathcal{B} X_{t-1} + \mathcal{I} u_t, \tag{15}$$

Turning to questions (i)-(iv), we may address (i) in a straightforward manner by an impulse response function analysis. This is achieved by focusing on the first *n* coordinates of the following representation of the impulse response function to a one standard deviation shock:

$$IRF^{(\mathbf{a})}(k) = \mathcal{B}^{k-1}\mathcal{I}\mathbf{a},\tag{16}$$

where k represents the period after the shock.

In order to tackle question (*ii*), I use the forecast error variance decomposition (FEVD) of equation (15). Let the mean square error of mistakenly forecasting  $X_{t+k}$  at period t be given by:

$$MSE(k) = \operatorname{Var} \left( X_{t+k} - \mathbb{E}_t \left( X_{t+k} \right) \right) = \sum_{i=1}^k \mathcal{B}^{i-1} \mathcal{I} \Sigma \mathcal{I} \mathcal{B}^{i-1}'$$

Additionally, notice that a matrix  $\mathcal{A}_0$  implementing an impulse vector **a** may be partitioned (without loss of generality) as  $\mathcal{A}_0 = [\mathbf{a} \ \mathbf{a}_2]$ , where  $\mathbf{a}_2$  represents the columns of  $\mathcal{A}_0$  other than **a**. In turn, this gives us the relationship  $\Sigma = \mathbf{a}\mathbf{a}' + \mathbf{a}_2\mathbf{a}'_2$  and the opportunity to pin down the component of MSE(k)due to the identified shock associated with impulse vector **a**:

$$MSE^{(a)}(k) = \sum_{i=1}^{k} \mathcal{B}^{i-1} \mathcal{I} (aa') \mathcal{I} \mathcal{B}^{i-1'}.$$

If we divide each entry of  $MSE^{(a)}(k)$  by MSE(k), define:

$$FEVD^{(a)}(k)_{i,j} = \frac{MSE^{(a)}(k)_{i,j}}{MSE(k)_{i,j}},$$
(17)

and focus on  $FEVD^{(a)}(k)_{i,i}$  for  $i \in \{1, ..., n\}$ , then, we have a measure of how much of the future expected variation of each variable is due to the shock identified by the impulse vector **a**.

Question (*iii*) is answered by a counterfactual exercise. More specifically, I use the historical decomposition proposed by Burbidge and Harrison (1985) to pin down how  $x_t$  would have evolved if only the identified shock had impulsed the economy during the sample period. In order to understand such procedure, it is useful to notice that, for a given set of estimated parameters  $(c_b, \mathcal{B}, \Sigma)$ , we can write the observed data  $\{X_t\}_{t=0}^T$  as:

$$X_t = \left(\sum_{i=0}^{t-1} \mathcal{B}^i\right) \mathcal{I} c_b + \mathcal{B}^t X_0 + \sum_{i=0}^{t-1} \mathcal{B}^i \mathcal{I} \hat{u}_{t-i},$$

where  $\{\hat{u}_t\}_{t=0}^T$  represents the reduced form residuals from the sample. Then, given an impulse vector **a**, we need to estimate the sample path of structural shocks  $\{\hat{\nu}_t^{(a)}\}_{t=0}^T$  associated with the estimated residuals  $\{\hat{u}_t\}_{t=0}^T$ . In other words, if we knew the entire matrix  $\mathcal{A}_0$  of which **a** is the j-th column,

we would like to know the scalars  $\hat{\nu}_t^{(a)}$  such that  $\hat{\nu}_t^{(a)} = (\mathcal{A}_0^{-1} \hat{u}_t)_j$ . For this, Uhlig (2005) shows that there is a non-zero unique vector  $\eta$  solving

$$(\Sigma - \mathbf{a}\mathbf{a}')\eta = 0 \quad \text{and} \quad \eta'\mathbf{a} = 1,$$
 (18)

such that  $\hat{\nu}_t^{(a)} = \eta' \hat{u}_t$ . Then, with a and  $\left\{ \hat{\nu}_t^{(a)} \right\}_{t=0}^T$ , we can find the counterfactual observations that would have happened if the economy had been impulsed only by the identified shock:

$$X_{t}^{(a)} = \underbrace{\left(\sum_{i=0}^{t-1} \mathcal{B}^{i}\right) \mathcal{I} c}_{\text{deterministic component}} + \underbrace{\sum_{i=0}^{t-1} \mathcal{B}^{i} \mathcal{I} a}_{\text{stochastic component}}, \qquad (19)$$

where the deterministic component is kept mostly for expositional purposes.

### F.2 Estimation

I use standard Bayesian procedures to estimate the VAR described in equation (8). Assuming that  $u_t \sim N(0, \Sigma)$  and letting  $x = [x_p \ x_{p+1} \dots \ x_T]'$ ,  $u = [u_p \ u_{p+1} \dots \ u_T]'$ ,  $z_t = [1 \ x'_{t-1} \dots \ x'_{t-p}]'$ ,  $z = [z_p \ z_{p+1} \dots z_T]'$ ,  $b = [c_b \ \mathcal{B}_1 \dots \ \mathcal{B}_p]'$ ,  $\mathbf{x} = \operatorname{vec}(x)$ ,  $\mathbf{u} = \operatorname{vec}(u)$ ,  $\mathbf{z} = I_n \otimes z$ ,  $\mathbf{b} = \operatorname{vec}(b)$ , we have the following Bayesian model:

$$\mathbf{x} = \mathbf{z}\mathbf{b} + \mathbf{u} \qquad \mathbf{u} \sim N\left(0, \Sigma \otimes I_{T-p}\right)$$
(20)

$$\mathbf{b}|\Sigma \sim N(\overline{\mathbf{b}}, \Sigma \otimes \Omega)$$
 (21)

$$\Sigma \sim IW(\Psi, d)$$
 (22)

where the last two equations represent the Normal-Inverse-Wishart prior on **b** and  $\Sigma$ . In order to avoid the problems associated with the use of a flat prior<sup>20</sup>, I use a version of the Minnesota prior designed to lead to closed-form solutions for the posteriors of **b** and  $\Sigma$ . On the Inverse-Wishart distribution (22), I set its degrees of freedom to a value as uninformative as possible: d = n + 2, the minimum value guaranteeing the existence of the prior mean of  $\Sigma$ . Regarding  $\Psi$ , I follow the standard practice in the literature by making it a diagonal matrix composed by the standard deviation of the residuals of a AR(1) fitted to each one of the time series in  $x_t$ . The parameters  $\overline{\mathbf{b}}$  and  $\Omega$  of the

 $<sup>^{20}</sup>$ It may lead to unacceptable estimators (Stein (1956)), bias towards stationarity and implausible forecasting power from initial conditions (Sims (2000)), and poor inference in large dimensional VARs (Sims (1980) and Litterman(1986)).

normal distribution (21) are chosen such that:

$$\mathbb{E}\left[\left(\mathcal{B}_{s}\right)_{ij}|\Sigma\right] = \begin{cases} 1 & \text{if } i = j \text{ and } s = 1\\ 0 & \text{otherwise,} \end{cases}$$
$$\operatorname{Cov}\left(\left(\mathcal{B}_{s}\right)_{ij}, \left(\mathcal{B}_{r}\right)_{hm}|\Sigma\right) = \begin{cases} \kappa^{2} \frac{1}{s^{2}} \frac{\Sigma_{ih}}{\Psi_{j}/(d-n-1)} & \text{if } m = j \text{ and } r = s\\ 0 & \text{otherwise.} \end{cases}$$

Essentially, this prior is centered on the assumption that each equation from (8) follows a random walk process, possibly with a drift. Regarding the precision of the prior, coefficients on more distant lags receive tighter priors around a zero mean. Finally, there is the parameter  $\kappa$  controlling the overall tightness of this prior. In order to choose such parameter, I use the approach derived by Giannonne et al (2012) by modeling  $\kappa$  also as a parameter subject to uncertainty, laying down its prior and simulating its posterior given the data<sup>21</sup>. Then, I fix its value on the mode of its posterior distribution and sample **b** and  $\Sigma$  from their posterior distributions:

$$\Sigma |\mathbf{x}, \mathbf{z} \sim IW \left( \Psi + \widehat{u}' \widehat{u} + \left( \widehat{b} - \overline{b} \right)' \Omega^{-1} \left( \widehat{b} - \overline{b} \right), T - p + d \right)$$
  
$$\mathbf{b} |\Sigma, \mathbf{x}, \mathbf{z} \sim N \left( \widehat{\mathbf{b}}, \Sigma \otimes \left( z' z + \Omega^{-1} \right)^{-1} \right)$$

where  $\overline{b}$  is such that  $\overline{\mathbf{b}} = \operatorname{vec}(\overline{b}), \ \widehat{b} = (z'z + \Omega^{-1})^{-1} (z'x + \Omega^{-1}\overline{b}), \ \widehat{u} = x - z\widehat{b} \text{ and } \widehat{\mathbf{b}} = \operatorname{vec}(\widehat{b}).$ 

In order to draw the impulse vectors, I use relationships (13) and (14). More precisely, for a given draw of b and  $\Sigma$ , I calculate the Choleski decomposition of  $\Sigma$  and define it as  $\tilde{\mathcal{A}}_0$ . If the objective is to use only one structural shock, I draw vectors **q** from the uniform distribution on unit length vectors until the impulse vector implied by **q** satisfies the desired identifying restriction (either Assumption 4.1 or 4.2). Then, with a draw  $(b, \Sigma, \mathbf{a})$ , I can calculate (16), (17) and (19). I repeat this procedure 10,000 times. If the objective is to use both structural shocks, then, the only difference is that instead of drawing unit length vectors, I draw matrices  $\mathcal{Q}$  from the uniform distribution on orthogonal matrices. Moreover, I draw  $\mathcal{Q}$ 's until it implies one impulse vector satisfying assumption 4.1 and another 4.2.

<sup>&</sup>lt;sup>21</sup>I am thankful for the authors for providing their code to implement this procedure.